TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE/

BACHELOR OF SCIENCE IN STATISTICS AND COMPUTER SCIENCE

AMA 4426: STOCHASTIC PROCESSES

END OF SEMESTER EXAMINATION

TIME: 2 HOURS

DATE: MAY 2017

Instructions to Candidates

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

This paper consists of 5 questions. Attempt ONE AND ANY TWO.

Do not write on the question paper.

QUESTION ONE (30 MARKS)

(a) Define the following:

(i) A stochastic process (2 marks)

(ii) A Bernoulli process (2 marks)

(b) Let Y have a geometric distribution given by

$$P(Y=k)=\begin{cases} q^k p; & k=0,1,2,3,\dots\dots \\ 0\ ; & elsewhere \end{cases}$$

Find (i) the probability generating function of Y (4 marks)

(ii) the mean and variance of Y (6 marks)

(c) . Let $\{X_n : n \geq 0\}$ be a Markov chain with three states 0,1,2 and transition probability matrix

$$\begin{pmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{pmatrix}$$

And the initial probability distribution $P(X_0 = i) = \begin{cases} \frac{1}{4}, i = 0 \\ \frac{1}{3}, i = 1 \\ \frac{5}{12}, i = 2 \end{cases}$

Find :

(i) $P(X_2 = 2 , \ X_1 = 1 / X_0 = 2)$ (3marks)

(ii) $P\ (X_1 = 1 / X_0 = 2)$ (1 mark)

(iii) $P\ (X_2 = 2 / X_1 = 1)$ (1 mark)

(iv) $P\ (X_3 = 1 , \ X_2 = 2 , X_1 = 1 , X_0 = 2)$ (3 marks)

(d). The joint distribution of two random variables X and Y is given by:

$$P_{jk} = P\{X = j , Y = k\} = \begin{cases} q^{j+k}p^2 , & j = 0,1,2, ... , k = 0,1,2, .., \quad p + q = 1 \\ 0 & otherwise \end{cases}$$

Obtain the:

(i). bivariate p.g.f of X and Y (4 marks)

(ii). P.g.f of X (2 marks)

(iii). P.g.f of X+Y (2 marks)

QUESTION TWO (20 MARKS)

(a) Define the following terms :

(i) Irreducible Markov chain (2 marks)

(ii) Persistent state (2 marks)

(iii) A periodic state (1 mark)

(iv) Ergodic state (1 mark)

(b). A markov chain with state space $\{E_1, E_2, E_3\}$ has the following probability transition matrix

$$\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$$

Classify the states of the process. (14 marks)

QUESTION THREE (20 MARKS)

Consider a population whose size at time t is Z(t) and let the probability that the population size is n be denoted by $P_n(t) = P\{Z(t) = n\}$ with $P_1(0) = 1$ *and* $P_n(0) = 0$, $n \neq 1$. Further let :

(i) The chance that an individual produces a new member in time t interval Δt be λΔt where λ is some constant be n. (5 marks)

(ii) The chance of an individual producing more than one member be 0(Δt) (i.e negligible).

(a) Show that $P_n(t) = e^{-\lambda t}\{1 - e^{-\lambda t}\}^{n-1}$, $n \neq 1$ (5 marks)

(b) Find the second raw moment of the process

(10 marks)

QUESTION FOUR (20 MARKS)

(a) Explain the following terms:

(i) A strictly stationary stochastic process (2 marks)

(ii) A covariance stationary process (2 marks)

(iii) An evolutionary process (2 marks)

b) (i) Write down the differential-difference equations for the Polya process. Hence obtain the probability generating function given that $P_n(0) = 1$ $when\ n = 0\ and\ P_n(0) = 0\ when\ n \neq 0$

(ii) Show that the Polya process is not covariance stationary.

(14 marks)

QUESTION FIVE (20 MARKS)

Consider the difference-differential equations for the Poisson process given by

$$P'_n(t) = \begin{cases} -\lambda P_n(t) + \lambda P_{n-1}(t) & : \ n \geq 1 \\ -\lambda P_0(t) & : \ n = 0 \end{cases}$$

With initial conditions $P_0(0) = 1\ when\ n = 0\ and\ P_n(0) = 0\ when\ n \neq 0$

(i) Find the solution of the equation. (10 marks)

(ii) Use Feller's method to find the mean and variance of the process.

(10 marks)