



TECHNICAL UNIVERSITY OF MOMBASA

A Centre of Excellence

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

**UNIVERSITY EXAMINATION FOR THE SECOND SEMESTER IN THE FOURTH
YEAR OF BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER
SCIENCE**

MAY 2016 SERIES EXAMINATION

UNIT CODE: AMA 4423

UNIT TITLE: PARTIAL DIFFERENTIAL EQUATIONS II

TIME ALLOWED: 2HOURS

PAPER B

Instructions to Candidates:

You should have the following for this examination

- *Answer Booklet*
- *Scientific Calculator*

This paper consists of **FIVE** questions and **TWO** sections **A** and **B**.

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

This paper consists of **THREE** printed pages.

SECTION A (COMPULSORY)

Question ONE (30 marks)

- a. Consider the following second order partial differential equation:-

$$3u_{xx} + 10xy u_{xy} + 3u_{yy} = 0$$

- (i) Classify it. (2 marks)
- (ii) Reduce to canonical form. (9 marks)
- (iii) Find the general solution in terms of arbitrary functions. (2 marks)

b. A string of length L is stretched between points $(0,0)$ and $(L,0)$ on the x axis. At time $t=0$ it has a shape given by $f(x)$, $0 \leq x \leq L$ and it is released from rest.

- i. Give the equation of a vibrating string described here (2 marks)
- ii. State the boundary and initial conditions associated with this problem (4 marks)
- iii. Find the displacement of the string at any latter time t . (11 marks)

SECTION B

Question TWO (20 marks)

a. Solve the Laplace's equation $\nabla^2 u = 0$ in two dimension which satisfies the conditions

$$u(0, y) = u(l, y) = u(x, 0) = 0 \text{ and}$$

$$u(x, a) = \sin \frac{n\pi x}{l}$$

by the method of separation of variables. (20 marks)

Question THREE (20 marks)

a. Show that in cylindrical coordinates r, θ, z defined by the relation

$$x = r \cos \theta, \quad y = r \sin \theta, \quad z = z, \text{ the Laplace's equation } \nabla^2 u = 0 \text{ takes the}$$

$$\text{form } \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} = 0 \quad (10 \text{ marks})$$

b. Classify and transform to canonical form $u_{xx} + x^2 u_{yy} = 0$ (10 marks)

Question FOUR (20 marks)

a. Obtain the general solution for $4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2$ (8 marks)

b. Solve by the method of characteristics $\frac{\partial v}{\partial t} + 3\frac{\partial v}{\partial x} = 0,$

$$v(x, 0) = \begin{cases} \frac{1}{2}x, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases} \quad (12 \text{ marks})$$

Question FIVE (20 marks)

a. Find the Fourier series expansion of $f(x) = x$ on $(-L, L)$ (8 marks)

b. Solve Laplace's equation inside a circle of radius a

$$\nabla^2 u = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = 0 \text{ subject to } u(a, \theta) = f(\theta) \quad (12 \text{ marks})$$