TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES
DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

DIPLOMA IN MECHANICAL, ELECTRICAL, BUILDING AND CIVIL ENGINEERING

YEAR III SEMESTER II

AMA 2251: ENGINEERING MATHEMATICS IV

END OF SEMESTER EXAMINATION

SERIES: DECEMBER 2016

TIME: 2HOURS

DATE: Pick Date December 2016

Instructions to Candidates

You should have the following for this examination
-*Answer Booklet, examination pass and student ID Mathematical table, calculator*
This paper consists of **FIVE** questions. Attempt question ONE (Compulsory) and any other TWO questions
Do not write on the question paper.

Question one (compulsory) (30MKS)

Question One (30 Marks)

a)

i) Determine $L\{e^{-2t}\sin 3t\}$ using Laplace transform tables *(2 marks)*

ii) Determine the Laplace transform of $f(x) = t^2$ from first principles *(6 marks)*

iii) Determine $L^{-1}\left\{\dfrac{4s-3}{s^2-4s-5}\right\}$ *(4 marks)*

b)

i) Given that $y=1$ when $x=2\frac{1}{6}$, determine the particular solution of

$(y^2-1)\frac{dy}{dx}=3y$ *(5 Marks)*

ii) Use the integrating factor to solve $\frac{dy}{d\theta}=\sec\theta+y\tan\theta$ given the boundary conditions $y=1$ when $\theta=0$ *(6 Marks)*

c) Determine the Fourier series expansion of the periodic function of period 1

$$f(x)=\begin{cases}\frac{1}{2}+x, & -\frac{1}{2}<x<0\\ \frac{1}{2}-x, & 0<x<\frac{1}{2}\end{cases}$$

(7 Marks)

Question Two (20 Marks)

a) Given the function $f(x)=x,\ \ 0<x<2\pi$, determine the Fourier series representing the function $f(x)$

(10 Marks)

b) Determine the Fourier series expansion of the periodic function of period 1, given the function

$$f(x)=\begin{cases}-1 & for \quad -\pi<x<\frac{-\pi}{2}\\ 0 & for \quad \frac{-\pi}{2}<x<\frac{\pi}{2}\\ 1 & for \quad \frac{\pi}{2}<x<\pi\end{cases}$$

(10 Marks)

<u>Question Three (20 Marks)</u>

a) A first order differential equation involving current i in a series $R-L$ circuit is given by: $\frac{di}{dt}+5i=\frac{E}{2}$ and $i=0$ and time $t=0$. Use Laplace transforms to solve for i when $E=50\sin 5t$ *(10 marks)*

b) Using Laplace transforms solve the following second order differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = e^{-x}\sin x, \qquad \text{where } y(0) = 0,\ y^{/}(0) = 1$$ (10 Marks)

Question Four (20 Marks)

a) Given that $7x(x-y)dy = 2(x^2 + 6xy - 5y^2)dx$ is homogeneous in x and y, solve the differential equation taking $x = 1$ when $y = 0$ (12 Marks)

b) The charge q in an electric circuit at time t satisfies the equation $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = E$, where L, R, C and E are constants. Solve the equation given $L = 2H$, $C = 200\times10^{-6}F$ and $E = 250V$ when $R = 200\Omega$ (8 Marks)

Question Five (20 Marks)

a) Show the following

$$\ell^{3t} = \frac{1}{s-3}$$, using the definition of Laplace transform *(3 marks)*

b) Determine $L^{-1}\left\{\frac{7s+13}{s(s^2+4s+13)}\right\}$

(9 marks)

c) Determine the general solution of $9\frac{d^2y}{dx^2} - 24\frac{dy}{dx} + 16y = 0$, then its particular solution given that $x = 0$, $y = \frac{dy}{dx} = 3$ (8 Marks)

Table of Laplace Transforms

	$f(t)=\mathcal{L}^{-1}\{F(s)\}$	$F(s)=\mathcal{L}\{f(t)\}$		$f(t)=\mathcal{L}^{-1}\{F(s)\}$	$F(s)=\mathcal{L}\{f(t)\}$
1.	1	$\frac{1}{s}$	2.	$\mathbf{e}^{at}$	$\frac{1}{s-a}$
3.	$t^n,\quad n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4.	$t^p,\ p>-1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n-\frac{1}{2}},\quad n=1,2,3,\ldots$	$\frac{1\cdot 3\cdot 5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$	8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10.	$t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11.	$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12.	$\sin(at)+at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13.	$\cos(at)-at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14.	$\cos(at)+at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15.	$\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\frac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
19.	$\mathbf{e}^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20.	$\mathbf{e}^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21.	$\mathbf{e}^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22.	$\mathbf{e}^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23.	$t^n\mathbf{e}^{at},\quad n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25.	$u_c(t)=u(t-c)$ Heaviside Function	$\frac{\mathbf{e}^{-cs}}{s}$	26.	$\delta(t-c)$ Dirac Delta Function	$\mathbf{e}^{-cs}$
27.	$u_c(t)f(t-c)$	$\mathbf{e}^{-cs}F(s)$	28.	$u_c(t)g(t)$	$\mathbf{e}^{-cs}\mathcal{L}\{g(t+c)\}$
29.	$\mathbf{e}^{ct}f(t)$	$F(s-c)$	30.	$t^n f(t),\quad n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31.	$\frac{1}{t}f(t)$	$\int_s^\infty F(u)\,du$	32.	$\int_0^t f(v)\,dv$	$\frac{F(s)}{s}$
33.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	34.	$f(t+T)=f(t)$	$\frac{\int_0^T \mathbf{e}^{-st}f(t)\,dt}{1-\mathbf{e}^{-sT}}$
35.	$f'(t)$	$sF(s)-f(0)$	36.	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37.	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$			