

## **TECHNICAL UNIVERSITY OF MOMBASA**

# FACULTY OF APPLIED AND HEALTH SCIENCES DEPARTMENT OF MATHEMATICS& PHYSICS

## **UNIVERSITY EXAMINATION FOR:**

## DIPLOMAIN MECHANICAL ENGINEERING AMA 2251: ENGINEERING MATHEMATICS IV

## END OF SEMESTER EXAMINATION

## SERIES:AUGUST2017

## TIME:2HOURS

DATE: Pick Date Aug 2017

Instructions to Candidates You should have the following for this examination -Answer Booklet, examination pass and student ID Scientific calculator This paper consists of FIVE questions. Attemptquestion ONE (Compulsory) and any other TWO questions Do not write on the question paper.

#### **Question One**

- a) The velocity of a body, V is equal to the rate of change of distance  $\frac{dx}{dt}$ . Determine the equation for x in terms of t, given V = u + at where u and a are constants and x = 0, when t = 0: (4 marks)
- b) Solve the differential equation:

$$9\frac{d^2y}{dy^2} = 12\frac{dy}{dt} + 4y = 0$$

Given

- y = 3 when t = 0 and  $\frac{dy}{dt} = 4$  when t = 0 (8 marks)
- c) Obtain the inverse of the laplace transform function

$$\left\{\frac{4S^2 - 5S + 6}{(S+1)(S^2 + 4)}\right\}$$
 (7 marks)

d) The periodic function y = f(x), of period  $2\pi$  is defined between x = 0 and  $x = \pi$  by the function value given in table 1. If the function is known to contain odd harmonics only:

(i) Show that 
$$a_0 = 0$$

- (ii) Determine  $a_1$
- (iii) Determine  $b_1$

Table 1							
$x^{0}$	00	30 <sup>0</sup>	$60^{0}$	90 <sup>0</sup>	120 <sup>0</sup>	150 <sup>0</sup>	$180^{0}$
у	0	8.0	11.5	6.0	4.0	5.4	0

(11 marks)

### **Question Two**

a) Solve the differential equation:

$$(y-x)\frac{dy}{dx} - \frac{y^2}{x} - y + \frac{X^2}{y}$$
 Given that  $x = 1$  when  $y = 3$  (8 marks)

b) An equation of Motion may be represented by the equation

$$\frac{dv}{dt} + Kv^2 = 0$$
 where

V is the velocity of a body traveling in a restraining medium.

Show that:

$$V = \frac{V_0}{1 + KtV_0}$$

Given that:

$$V = V_0 \text{ when } t = 0 \tag{5 marks}$$

c) Solve the differential equation:

$$x \frac{dy}{dx} = y + x^2 - 2x$$
 given  $X = 1$  when  $y = 3$  (7 marks)

### **Question Three**

a) Solve the differential equation:

$$6\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 4y = 0, \quad \text{Given } y = 11 \text{ when } = 0 \text{ and } \frac{dy}{dx} = 0 \text{ when } x = 0 \quad (8 \text{ marks})$$

b) Solve the differential equation:

$$15 \frac{d^2 y}{dx^2} - 2\frac{dy}{dx} - y = 3X + 65SinX$$
 (12 marks)

### **Question Four**

a) Obtain from first principles:

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	(i)	$L{t}$	(4 marks)
	(ii)	$L\{e^{at}\}$	(3 marks)
b)	Obtair	using the appropriate shift theorem the laplace transform of	
	(i)	$\{t \sin 2t\}$	(3 marks)
	(ii)	$\left\{e^{-3t}Sin2t\right\}$	(3 marks)

c) Solve the equation 
$$\frac{dx}{dt} + 2x = 10e^{3t}$$
 given that at  $t = 0$ ;  $X = 6$  (7 marks)

### **Question Five**

The values of f(x), a periodic function of period  $2\pi$ , at intervals of  $30^{\circ}$  from  $X = 0^{\circ}$  and  $X = 360^{\circ}$  are as given in table 1.

#### Table 1

$X^{0}$	00	300	60 <sup>0</sup>	90 <sup>0</sup>	1200	$150^{0}$	$180^{0}$	2100	$240^{\circ}$	$270^{\circ}$	300 <sup>0</sup>	330 <sup>0</sup>	360 <sup>0</sup>
f(x)	) 1.4	1.6	2.0	2.1	1.9	1.1	0.4	0.4	0.7	0.6	0.5	1.0	1.4

Determine the corresponding Fourier series for f(x) up to the second harmonics. (20 marks)