# UNIVERSITY EXAMINATION FOR: 

END OF SEMESTER EXAMINATION<br>SERIES: December 2016<br>TIME: two HOURS

## Instructions to Candidates

You should have the following for this examination
Answer Booklet, examination pass and student ID, Scientific Calculator \& No Mobile Phone.
This paper consists of five questions. Attempt Question One COMPULSORY and any other TWO questions.
Maximum marks for each part of a question are as shown.
This paper consists of THREE printed pages
Do not write on the question paper.

## QUESTION ONE (COMPULSORY)

(a) Determine from first, principles the derinative $f(x)=\frac{1}{5 x+3}$
(b) Given $u=x^{2} y+y$ find du
(c) Given that $f(x)=x^{2}$ express as simply as possible

$$
\frac{f(a+h)-f(a)}{h}(h+0)
$$

(d) If $x^{3}+y^{3}+3 x y^{2}=8$ find $\frac{d y}{d x}$
(a) Evaluate $\int \sqrt{ }\left(a^{2}-x^{2}\right) d x$ by putting $x=a \sin \Theta$
(b) Find the value of

$$
\operatorname{Lim}_{x-} \frac{3 x^{2}+4 x-2}{5 x^{2}-x r 6}
$$

$$
\text { by putting } \mathrm{x}=\frac{1}{\mathrm{~h}} \text { and } \mathrm{h}-0 \text {. }
$$

(c) Determine $\int \frac{7 x d x}{\sqrt{ }\left(8 x^{2}+4\right)}$

## QUESTION TWO:

(a) Find the gradient at the point $(1,2)$ on the curve $y=x^{3}+3 x^{2}-x-1$
(b) A box with sides of length $\mathrm{x}, \mathrm{y}, \mathrm{zmm}$ is expanding along the $x$ and $y$ sides at a rate of 2 and 3 mm per second but contracting along the z side at a rate of 4 mm per second. Find the rate of change of volume when $x=y=10 \mathrm{~mm}, \mathrm{z}=20 \mathrm{~mm}$
(c) If $x=t^{3}+t^{2}, y=t^{2}+t$. Find dy in terms of $t$. dx
(d) Sketch the curves $y=4-x^{2}$ and $y=x^{2}-2 x$, then find the area enclosed between the two curves

## QUESTION THREE:

(a) Find the maximum and minimum of the function

$$
\begin{equation*}
y=x^{3}+6 x^{2}-36 x+5 \tag{6marks}
\end{equation*}
$$

(b) Find the equation of the normal to the curve $y=\left(x^{2}+x+1\right)(x-3)$
at the point where it cuts the $\mathrm{x}-$ axis. (Take $\mathrm{x}^{2}+\mathrm{x}+1$ )
as having no real roots)
(c) A pin moves along a straight guide so that is velocity $\mathrm{v}(\mathrm{cm} / \mathrm{s})$ when it is distance $x(\mathrm{~cm})$ from the beginning of the guide at time $t(s)$ is as given in the table below

| $\mathrm{t}(\mathrm{s})$ | 0 | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{~V}(\mathrm{~cm} / \mathrm{s})$ | 0 | 4.00 | 7.94 | 11.6 | 14.97 | 17.39 | 18.25 | 16.08 | 0 |

Apply Simpsons rule using 8 intervals, to find the approximate total distance travelled by the pin between $\mathrm{t}=0$ and $\mathrm{t}=4$

## QUESTION FOUR:

(a) Given $\operatorname{Cos}^{4} \Theta=1 / 4(1+\operatorname{Cos} 2 \theta)^{2}$

Evaluate $\int \operatorname{Cos} 4 \Theta \mathrm{~d} \Theta$
(b) The area of the segment cut off by $Y=5$ from the curve $y=x^{2}+1$ is rotated about the $x-$ axis. Find the volume generated
(c) Show that $\left.V=\left(\mathrm{Ar}^{\mathrm{n}}+\underline{\mathrm{B}}\right) \operatorname{r}\right) \cos (\mathrm{n} \Theta-)$

Satisfies the equation

$$
\begin{equation*}
\frac{d^{2} y}{d^{2}}+\frac{1}{r d r} d v+\frac{1}{r^{2}} \frac{d^{2} y}{d \Theta^{2}}=0 \tag{7marks}
\end{equation*}
$$

## QUESTION FIVE:

(a) Evaluate (i) $\quad \int \mathrm{x} \sqrt{ }(3 \mathrm{x}-1) \mathrm{dx}$ by substitution
$\int \frac{x+1}{x^{2}-3 x+2} d x \quad$ by partial fractions (5 marks)
(b) Given cash $\mathrm{x}=1 / 2\left(\mathrm{e}^{\mathrm{x}}+\mathrm{e}^{-\mathrm{x}}\right)$ and

Shinex $=1 / 2\left(e^{x}-e^{-x}\right)$
Show that for $\tanh ^{-1} \mathrm{x},=1 / 2 \ln \left(\frac{1+\mathrm{x}}{1-\mathrm{x}}\right)$
(6 marks)
(c) Evaluate

$$
\begin{equation*}
\mathrm{I} \quad=\int_{1}^{3} \int_{1}{ }^{1} \int_{0}^{2}(\mathrm{x}+2 \mathrm{y}-\mathrm{z}) \mathrm{dxdydz} \tag{5marks}
\end{equation*}
$$

