TECHNICAL UNIVERSITY OF MOMBASA

A Centre of Excellence

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

DECEMBER 2016 SERIES EXAMINATION

UNIT CODE: EMG 2414/ AMA4410/ AMA4306
UNIT NAME: NUMERICAL METHODS FOR ENGINEERS

SPECIAL/SUPPLIMENTARY EXAMINATION

TIME ALLOWED: 2HOURS

INSTRUCTIONTO CANDIDATES:

You should have the following for this examination

- Mathematical tables
- Scientific Calculator

This paper consists of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

QUESTION ONE (30 MARKS) COMPULSORY

a. If $AA^{-1} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Find $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ (2 marks)

b. Using **Romberg's** integration method, find the value if starting with trapezoidal rule for the given tabular values below given $h = 0.8\ for\ \frac{h}{2}, \frac{h}{4}, \frac{h}{8}$ (7 marks)

x	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
Y=f(x)	1.543	1.669	1.811	1.971	2.151	2.352	2.577	2.8228	3.107

c. Evaluate

$\int_0^1 \frac{1}{1+x} dx$ Using Trapezoidal rule for n=1 and state the error bound (4 marks)

d. Find $\lambda \ for \begin{pmatrix} \lambda & \lambda \\ 3 & \lambda - 2 \end{pmatrix} = 0$ (3 marks)

e. Consider the system of equations

$\frac{dx}{dt} = x - y - z$

$\frac{dy}{dt} = y + 3z$

$\frac{dz}{dt} = 3y + z$

i. Write in the form
ii. Find eigen values and eigen vectors (4 marks)
iii. Prove that the solutions are independent (2 marks)
iv. Hence, write general solution (1 marks)

f. Determine Fourier transform of $f(t) = e^{kt}, 0 \leq t \leq \infty$ (4 marks)

g. Given

$$\frac{dy}{dt} = \frac{y - t}{y + t}$$

with the initial condition $y = 1$ at $t = 0$

Find y approximately at $x = 0.1$, in five steps using Euler's method. (3 marks)

QUESTION TWO (20 MARKS)

a. Define linear independence of functions (2 marks)

b. Obtain Picard's second approximate solution of the initial value problem $\frac{dy}{dx} = \frac{x^2}{y^2+1}, y(0) = 0$ (4 marks)

c. Find the inverse of the matrix A by Gaussian method

$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 5 & 5 & 1 \end{pmatrix}$ (6 marks)

d. Solve the initial value problem

$\frac{dy}{dx} = x^2 - y \qquad y(0) = 1$

With h=0.2 on the interval (0, 0.4) using the forth order Runge- Kutta Method.

With $y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(t_n, y_n)$

$$k_2 = hf(t_n + \frac{h}{2}, y_n + \frac{k_1}{2})$$
$$k_3 = hf(t_n + \frac{h}{2}, y_n + \frac{k_2}{2})$$
$$k_4 = hf(t_n + h, y_n + k_3)$$
(8 marks)

QUESTION THREE (20MARKS)

a. Determine the ad joint of a matrix A if

$$A = \begin{pmatrix} 1 & 5 & -2 \\ 3 & -1 & 4 \\ -3 & 6 & -7 \end{pmatrix}$$

Hence compute A^{-1} the inverse of the matrix (6 marks)

b. I) Find the determinant of the following matrix

$$\begin{pmatrix} -4 & 9 & 2 \\ 5 & 6 & -1 \\ 3 & 2 & 7 \end{pmatrix}$$

(3 marks)

ii) For the system of equations given below, form the augmented matrix hence solve for the three unknowns using Gaussian elimination method (5 marks)

$$2x + 4y + 7z = 82$$
$$6x - 3y + z = 11$$
$$x + 2y - 5z = -27$$

c. Compute the integral using Simpson's 1/3 rule taking h=0.125 (6 marks)

$$I = \sqrt{\frac{2}{\Pi}} \int_0^1 e^{\frac{-x^2}{2}} dx$$

QUESTION FOUR (20 MARKS)

a. Using Crout's reduction, decompose the matrix A

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 5 & 4 \\ 3 & 1 & 3 \end{pmatrix}$$

In to $[L]$ $[U]$ form and hence solve the system of equations

$$x + y + z = 7$$
$$3x + 5y + 4z = 24$$
$$3x + y + 3z = 16$$

(7 marks)

b. Use Gauss Legendre quadrature formula to compute the integral

$I = \int_5^{12} \frac{dx}{x}$ for n=3 in the interval (-1, 1) (5 marks)

c. Given that $A = \begin{pmatrix} 1+j & 2j \\ -3j & 1-4j \end{pmatrix}$ and that $j^2 = -1 = j.j$. Determine det A (2 marks)

d) If A(t) = $\begin{pmatrix} e^t & 2e^{-t} & e^{2t} \\ 2e^t & e^{-t} & -e^{2t} \\ e^t & 3e^{-t} & 2e^t \end{pmatrix}$

i) Find $\frac{dA}{dt}$ (2 marks)

ii) Determine $\int_0^1 A(t)dt$ (4 marks)

QUESTION FIVE (20 MARKS)

a. Employ Taylors method to obtain an approximate value of y at x=0.2 for the differential equation given below and compare the results with the exact solution $\frac{dy}{dx} = 2y + 3e^x, y(0) = 0$ (5 marks)

b. Given $\frac{dy}{dx} = 1 + y^2$ approximate $y(0.8)$ using the Milne's Predictor Corrector method if

x	0.0	0.2	0.4	0.6
y	0	0.2027	0.4228	0.6841

(7 marks)

c. Consider the following system of linear equations.

$x + y + z = 4$ $2x -$
$3y + 4z = 33$ $3x - 2y -$
$2z = 2$

Apply Crammer's rule to determine x, y and z

(6 marks)

d. Given that

$$A = \begin{pmatrix} 3 & 2-i \\ 4+3i & -5+2i \end{pmatrix}$$

Find the adjoint of A (2 marks)

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