TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Engineering and Technology
Department of Mechanical & Automotive Engineering
UNIVERSITY EXAMINATION FOR:
BSc. Mechanical Engineering
EMG 2410 : Control Engineering II
END OF SEMESTER EXAMINATION
SERIES: DECEMBER 2016
TIME: 2 HOURS
DATE: 15 Dec 2016

Instructions to Candidates

You should have the following for this examination

- *Answer booklet*
- *Non-Programmable scientific calculator*

This paper consists of **FIVE** questions. Attempt question **ONE** and any other **TWO** questions.

Maximum marks for each part of a question are as shown.

Do not write on the question paper.

Question ONE (Compulsory)

a. Most controllers used in industry are the *three term* or *PID controllers*. PID stands for:

- **P**roportional term in the controller.
- **I**ntegral term in the controller.
- **D**erivative term in the controller.

Describe the control action of each term in the controller and define their mathematical model. **(6 marks)**

b. Given a plant transfer function as follows, $Y(s)/U(s) = 10/(s+1)(s+2)$, design state feedback for the plant represented in cascade form to yield a 15% overshoot with a settling time of 0.5 second.

(8 marks)

- Maximum overshoot M_p

$$M_p = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}$$

- Settling time t_s of within 5%

$$t_s = \frac{3}{\xi\omega_n}$$

c. Consider the closed loop system with a unity feedback system as shown in Figure Q1c.

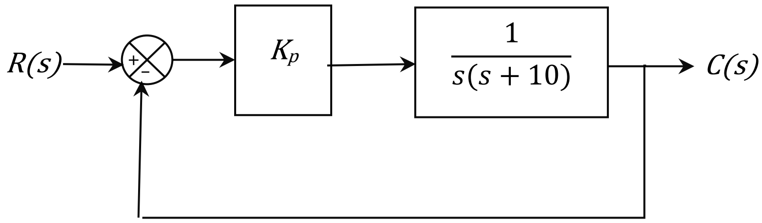

Figure Q1c

For reference input, $r(t)=0.2t$, Determine the exact value of the proportional gain K_p so that the system will be critically damped and with a steady-state error given by, $e_{ss} \leq 0.02$.

(8 marks)

d. The Figure Q1d shows a typical closed-loop system with the process transfer function $G_1(s)$ and the controller transfer function $K(s)$

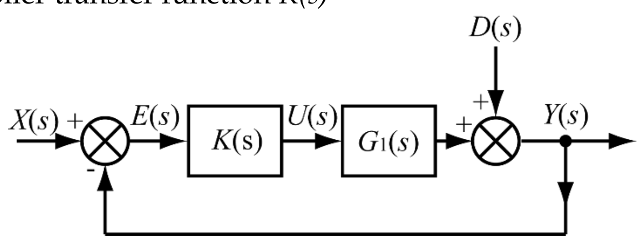

Figure Q1d

Determine the closed-loop transfer function, $G(s)$, and the sensitivity (disturbance rejection) transfer function, $S(s)$ for the following systems.

i. $K(s) = K_P + K_d s,\ G_1(s) = \dfrac{32}{s^2 + 2.4s + 16}$

ii. $K(s) = K_P + \dfrac{K_i}{s},\ G_1(s) = \dfrac{12}{s + 10}$ **(8 marks)**

Question TWO

Figure Q2 shows the block diagram of a second-order system with a proportional controller with gain, K, and a unity negative feedback loop.

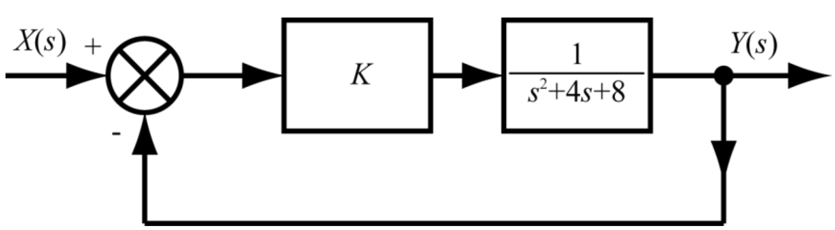

Figure Q2

a. Sketch the root locus diagram for this system showing how the poles vary as K increases from a value of zero. **(5 marks)**

b. If a unit step input is applied at $x(t)$ and a value of $K = 10$ is used calculate the following:

   i. the steady-state error ,
   ii. and the time for the system response to fall and remain within 2% of the steady-state value assuming that this is calculated using:
      - Settling time t_s of within 2%

$$t_s = \frac{3.9}{\xi \omega_n}$$

(5 marks)

c. If a block with transfer function $\frac{1}{s}$ were introduced between the controller and second-order system blocks in Figure Q2 state, giving reasons, how this would affect the following, assuming the value of $K = 10$ and a unit step input were still used:

   i. the steady-state error,
   ii. the time for the system response to fall and remain within 2% of the steady-state value given the information that one of the closed-loop poles is located at $s = -2.4$
   iii. and the relative stability of the system compared to the system of parts a and b.

(10 marks)

Question THREE

a. In the design of state variable feedback systems, the conditions of controllability and observability may govern the existence of a complete solution to the control system design problem. Define the following terms:

   i. Controllability
   ii. Observability **(4 marks)**

b. Consider a transfer function of a system as shown in Figure Q3

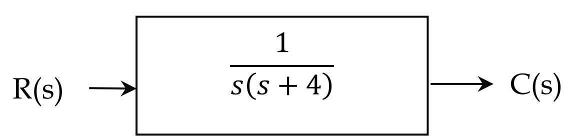

Figure Q3

   i. Find the state equation and output equation for the phase variable representation of the transfer function.

   ii. Evaluate the coefficients of the state feedback gain matrix such that the closed-loop poles have the values -2, -2. **(16 marks)**

Question FOUR

a. Let the reference input to the system as shown in Figure Q4 below be a unit ramp and determine the steady-state error for the controller $K(s)$ being:

   i. A PD controller,

ii. A PI controller **(10 marks)**

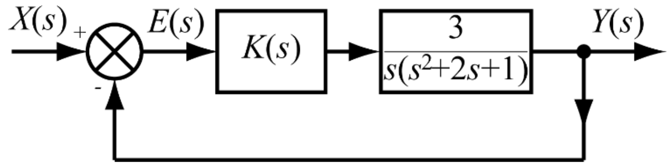

Figure Q4

b. For the general closed-loop system as shown in the previous question (a) but with $G_1(s) = \frac{2.5}{s^2 + 0.1s + 0.25}$ calculate the gains K_p and K_d in a PD controller which give a closed-loop damping ratio of 0.5 and a closed-loop natural frequency of 4 rad/s.
(10 marks)

Question FIVE

a. The plant manager responsible for a chemical plant is unhappy with the performance of one of the processes. When a 10°C reference input step change is applied to the process this only results in a 5°C output temperature change. The time constant of the system is 20 minutes.

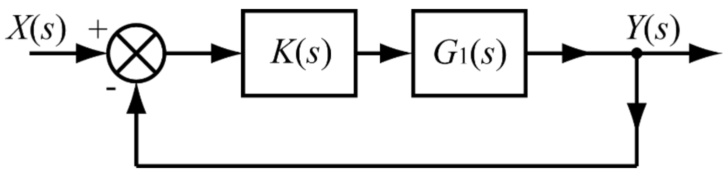

Figure Q5

i. Assuming that the process can be represented by a first-order model, given the above information, specify the system transfer function, $G_1(s)$, relating the output temperature to the input temperature.

ii. Sketch the output response of the process to the 10°C reference input step change, clearly indicating the desired steady-state value, the actual steady-state value and the relevance of the time constant.

iii. Determine the steady-state error of the process.

iv. Initially, the manager is only concerned with the slow system response to the step change and desires that the time-constant of the system should be reduced from 20 minutes to 5 minutes. The closed-loop control system is as shown in Figure Q5 where $G_1(s)$ is the transfer function of the process and $K(s)$ is the controller transfer function. The manager decides to use proportional control to achieve this one design requirement. Show that the required value for the proportional controller gain to satisfy the design requirement is $K_p = 6$.

v. The plant manager subsequently decides to add an integral control component to the proportional component to give perfect reference tracking (zero steady-state error). It is decided that a closed-loop system damping ratio of 1 is desirable. Calculate the value of

K_i, the integral controller gain, which satisfies the new design requirement, assuming that the proportional gain is kept at a value of $K_p = 6$.

vi. Comment upon the potential control benefits, if any, of including a derivative block in the controller for the first-order process discussed above. **(20 marks)**