# TECHNICAL UNIVERSITY OF MOMBASA 

FACULTY OF APPLIED AND HEALTH SCIENCES
DEPARTMENT OF MATHEMATICS \& PHYSICS

UNIVERSITY EXAMINATION FOR: DIPLOMA IN MECHANICAL ENGINEERING AMA 2251: ENGINEERING MATHEMATICS IV<br>END OF SEMESTER EXAMINATION SERIES: DECEMBER 2016<br>TIME: 2HOURS<br>DATE: Pick Date Dec 2016

## Instructions to Candidates

You should have the following for this examination
-Answer Booklet, examination pass and student ID
This paper consists of FIVE questions. Attempt question ONE (Compulsory) and any other TWO questions.
Do not write on the question paper.
Q. 1 a) A tank contains 50 litres of salt solution containing 1 Kg of salt. Another salt solution with 0.2 Kg of salt per litre runs in at 3 litres $/ \mathrm{min}$ and the mixture runs out at the same rate.

Determine the amount of salt in the tank
(i) After t - minutes
(ii) After 15 - minutes
b) Solve for y in the differential equation

$$
\begin{equation*}
x y=\left(1+x^{2}\right) \frac{d y}{d x} \tag{6marks}
\end{equation*}
$$

c) Obtain using the First shift theorem $L\left\{e^{-3 t} \operatorname{Sin} 2 t\right\}$
d) Obtain $L^{-1}\left\{\frac{s^{2}-15 s+4}{(s+2)(s-3)^{2}}\right\}$
e) Given the following set of values

| Xo | 0 | 30 | 60 | 90 | 120 | 150 | 180 | 210 | 240 | 270 | 300 | 330 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{f}(\mathrm{x})$ | 1.8 | 1.7 | 1.5 | 1.0 | 0.6 | 0.4 | 0.5 | 1.0 | 1.6 | 2.0 | 2.1 | 1.9 |

Determine
(i) The constant term $\mathrm{a}_{0}$ in the Fourler series
(ii) The coefficient of $f(x) \operatorname{Cos} x, a_{1}$.
Q. 2 a) The rate at which a body cools is given by the equation $\underline{d \theta}=-k \theta$, where $\theta$ is the dt temperature of the body above its surrounding and k is a constant. Solve the Equation for $\theta$ given that at $t=0, \theta=\theta_{\text {o }}$
b) Solve the differential equation

$$
(y-x) \frac{d y}{d x}=\frac{y^{2}}{x}-y+\frac{x^{2}}{y}
$$

given that $\mathrm{x}=-1$ when $\mathrm{y}=3$.
c) Determine the particular solution of the differential equation $\frac{d y}{d x}+2 x=y$, given that $\mathrm{x}=0$ and $\mathrm{y}=2$.
(3 marks)
Q. 3 a) Obtain from first principles
(i) $L\left\{t^{2}\right\}$
(ii) $L\left\{\frac{d^{2} x}{d t^{2}}\right\}$
(10 marks)
b) Determine $L^{-1}\left\{\frac{5 s+1}{S 2-s-12}\right\}$
c) Use Laplace transforms to solve the Differential equation.

$$
\frac{d^{2} x}{d t^{2}}-3 \frac{d x}{d t}+2 x=0 \text { given when } \mathrm{t}=0, \mathrm{x}=4 \text { and } \mathrm{dx} / \mathrm{dt}=3
$$

(10 marks)
Q. 5 Given the corresponding values for $\mathrm{x}^{\circ}$ and $\mathrm{f}(\mathrm{x})$ are as tabulated

| $\mathrm{X}^{0}$ | 0 | 30 | 60 | 90 | 120 | 150 | 180 | 210 | 240 | 270 | 300 | 330 | 360 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{f}(\mathrm{x})$ | 1.4 | 1.6 | 2.0 | 2.1 | 1.9 | 1.1 | 0.4 | 0.4 | 0.7 | 0.6 | 0.5 | 1.0 | - |

Determine the Fourier series up to and including the second harmonic.
(20 marks)
Q. 2 a) Determine the particular solution to the differential equation

$$
4 \frac{d^{2} y}{d x^{2}}-12 \frac{d y}{d x}+9 y=0
$$

Given when $x=0, y=2$ and $\frac{d y}{d x}=4$
(7 marks)
b) Determine the particular solution of the equation

$$
3 \frac{d^{2} x}{d t^{2}}+\frac{d y}{d x}-4 y=e^{-3 x}
$$

Given the boundary conditions that when $x=0, y=3 / 5$ and $\underline{d y}=-64 / 5 \quad(13$ Marks) dx

