

### **TECHNICAL UNIVERSITY OF MOMBASA**

## FACULTY OF APPLIED AND HEALTH SCIENCES DEPARTMENT OF MATHEMATICS & PHYSICS **UNIVERSITY EXAMINATION FOR:** DIPLOMA IN MECHANICAL, ELECTRICAL, BUILDING AND CIVIL ENGINEERING YEAR III SEMESTER II AMA 2351: ENGINEERING MATHEMATICS VI END OF SEMESTER EXAMINATION **SERIES:** DEC 2016 **TIME:** 2HOURS DATE: Pick Date DEC 2016

#### **Instructions to Candidates**

You should have the following for this examination *Answer Booklet, examination pass and student I Mathematical table, calculator* This paper consists of **FIVE** questions. Attempt question **ONE** (Compulsory) and any other **TWO** questions.

#### Do not write on the question paper.

#### **Question one (compulsory)**

1. a) Given the function 
$$u(x,y) = x^2 - y^2 + x$$
  
i) Show that  $u(x,y) = x^2 - y^2 + x$  is harmonic. (3marks)

ii) Determine a harmonic conjugate function v(x, y) such that f(z) = u + jv

is analytic.

(4marks)

- b) Show that  $f(z) = z^3$  is analytic everywhere in the entire region in the z-plane. (5marks)
- c) Using Newton-Gregory interpolation formula, evaluate f(1.03) correct to six

decimal places.

х	1	1.05	1.10	1.15	1.20
F (x)	1	1.257625	1.531	1.820875	2.128

(8marks)

(6marks)

d) i) Use double integrals to find the area enclosed  $y = x^3 + 4x$ , y = o, x = 0 and x = 4 (6marks)

ii) Evaluate 
$$\int_{0}^{1} dx \int_{0}^{x} dy \int_{0}^{y} dz$$
 (4marks)

#### **QUESTION TWO**

- a) Use Newton-Gregory forward difference formula and the data below to compute
  - i) f(17.5) 6marks
  - ii) f(35.5) Correct to three decimal places

х	10	15	20	25	30	35	40
F(x)	1736	2588	3420	4226	5000	5736	6428

b)

(i) Given that  $x_n$  is an approximation to the root of the equation  $x^3 - 4x^2 + 4 = 0$ 

Show using Newton – Raphson iterative method that a better approximation is

given by 
$$x_{n+1} = \frac{2x_n^3 - 4x_n^2 - 4}{3x_n^2 - 8x_n}$$

ii) Hence by taking  $x_{\circ} = 0.5$ , find the root of the equation correct to SIX d.p (8marks)

#### **QUESTION THREE**

5. a) State Greens Theorem

# (b) Using Greens theorem evaluate $\oint_c \{(x^2 + y^2)dx + (x + 2y)dy\}$ taken round the boundary

curve c defined by

$$y = 0$$
 $0 \le x \le 2$ 
 $x^2 + y^2 = 4$ ,
  $0 \le x \le 2$ 
 $x = 0$ 
 $0 \le y \le 2$ 

 (10marks)

(3mraks)

c) A function 
$$f(x, y) = x^2 + y$$
 is defined on the rectangular region  $0 \le x \le 1, 1 \le y \le 2$ 

show that 
$$\int_{1}^{2} \int_{0}^{1} f(x,y) dx dy = \int_{1}^{2} dy \int_{0}^{1} f(x,y) dx = \int_{0}^{1} dx \int_{1}^{2} f(x,y) dy = \frac{11}{6}$$
 (7marks)

#### **QUESTION FOUR**

a) Find the volume enclosed by the curve  $x^2 + y^2 = 16$ , and the planes

$$z = 0 \text{ and } z = 5 - x \tag{8marks}$$

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I) 
$$\int_{1}^{3} \int_{0}^{\ln y} dx dy$$
 (5marks)

II) 
$$\int_{0}^{\frac{\pi}{2}} \int_{0}^{x} x \sin y dy dx$$
 (7marks)

#### **QUESTION FIVE**

a (i)	State the Cauchy-Riemann Equations	1mark
(ii)	Verify that the following function satisfies the Cauchy-Riemann equations and	

express the derivative of W = f (z) as a function of Z

$$W = f(z) = (x^{3} - 3xy^{2} + y) + j(3x^{2}y - y^{3} - x)$$
 8marks

b) The circle |z| = 2 in the Z-plane is mapped onto the w-plane under a transformation

$$w = \frac{1}{z-1}$$
. Determine the image of the circle in the w plane (11marks)