TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Engineering and Technology

Department of Electrical and Electronic Engineering

UNIVERSITY EXAMINATION FOR:

Higher Diploma in Electrical Engineering

SUPPLEMENTARY EXAMINATION

AMA 3151 ENGINEERING MATHEMATICS II

END OF SEMESTER EXAMINATION

SERIES: DEC 2017

TIME: 2HOURS

DATE: DEC 2017

Instructions to Candidates

You should have the following for this examination

-*Answer Booklet, examination pass and student I Mathematical table, calculator*

This paper consists of **FIVE** questions. Attempt question **ONE** (Compulsory) and any other **TWO** questions.

Do not write on the question paper.

QUESTION ONE (compulsory)

1a) (i) State the Cauchy – Riemann equations. (2marks)

(b) Express $f(z) = j - \frac{1}{\pi}\ln z$ in the form u + j v. Hence, show that

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0 \text{ and } \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \text{ is harmonic}$$ (10marks)

c) The circle $|z| = 4$ is described in the z plane in anti-clockwise manner. Obtain its image in the W plane under the transformation $W = \frac{z+1}{z-2}$ and state the direction of development

i. Sketch the circle in the Z-plane and W-plane.

ii. II) Determine the centre and radius of the resulting circle in the W-plane. (11 marks)

d) Expand $f(z) = \sin z$ in Taylor series about $z = \frac{\pi}{4}$ (7marks)

QUESTION TWO

(a) The instantaneous current I passing through a circuit of resistance R and inductance L satisfies the differential equation. $L\frac{di}{dt} + Ri = V_o \cos \omega t$

Where t is time and V_o and ω are constant. Show that

$$i = \frac{V_o}{\omega^2 L^2 + R^2}\{L\omega \sin \omega t + R\cos \omega t\} + Ce^{-\frac{R}{L}t}$$ (9marks)

b) State the Legendre Linear Equation (1mark)

c) Solve completely the differential equation by using the substitution $e^Z = (2x-1)$

$$(x+2)^2\frac{d^2y}{dx^2} - (x+2)\frac{dy}{dx} + y = 3x + 4.$$ (10 marks)

QUESTION THREE

a) Solve the difference equation $y_{n+2} - 2y_{n+1} + y_n = 2^n \cdot n^2$ (6marks)

b) Find the Residue of $f(z) = \dfrac{z}{z^2+1}$ (6marks)

c) Evaluate $\int_c \dfrac{2z-1}{z(z+1)}\, dz$ using residue theorem where c is a circle $|z| = 2$ (8marks)

QUESTION FOUR

(a) Determine where the function $W = z^2 - 4$ fails to be regular (3marks)

b) Solve the following differential equations.

c) $\dfrac{d^2x}{dt^2} - 6\dfrac{dx}{dt} + 10x = 20 - e^{2t}$ $when\, t = 0, x = 4, \dfrac{dx}{dt} = \dfrac{25}{2}$ (8marks)

c Given that $u = x^2 - y^2 + e^x \cos y + 8$

i) Show that U is harmonic

ii) Find the function V such that $f(z) = u + jv$ is analytic where U is as in (i)

(9marks)

QUESTION FIVE

(a) State Cauchy linear equation.

(b) Use the substitution $x = e^t$ to express the differential equation.

$x^2 \dfrac{d^2y}{dx^2} + 5x\dfrac{dy}{dx} + 3y = \left(1 + \dfrac{1}{x}\right) lnx$ in the form $a\dfrac{d^2y}{dt^2} + b\dfrac{dy}{dt} + cy = f(t)$

Where a, b and c are constants. Hence, solve the differential equation. (20 marks)