TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Business and Social Studies

DEPARTMENT OF BUSINESS STUDIES

UNIVERSITY EXAMINATIONS FOR DEGREE IN

BACHELOR OF BUSINESS ADMINISTRATION

BACHELOR OF COMMERCE

BMS 4102: MANAGEMENT MATHEMATICS II

END OF SEMESTER EXAMINATIONS

SERIES: DECEMBER 2014

TIME: 2 HOURS

INSTRUCTIONS:

- Answer Question **ONE (Compulsory)** and any other **TWO** questions.
- Do not write on the question paper

This paper consists of Six printed pages

QUESTION 1 (Compulsory)

a) Multiply the following matrices:

$$\begin{bmatrix} 1 & 0 & 2 & 3 \\ 2 & -1 & 0 & 3 \\ 3 & 2 & -2 & 0 \end{bmatrix} x \begin{bmatrix} 2 & 1 \\ 0 & 2 \\ 1 & 4 \\ 2 & 0 \end{bmatrix}$$ **(2 marks)**

b) $$\begin{bmatrix} 3 & 2 & 4 \\ -1 & 0 & 1 \end{bmatrix} x \begin{bmatrix} 2 & -1 \\ 4 & -2 \\ 3 & 0 \end{bmatrix}$$ **(2 marks)**

c) Find the determinant of the following matrices:

$$A = \begin{pmatrix} 2 & 4 & 7 \\ -1 & 3 & 2 \\ 4 & -2 & 0 \end{pmatrix}$$ **(2 marks)**

d) Find the inverse matrix A^{-1} for $\begin{bmatrix} 20 & 5 \\ 6 & 2 \end{bmatrix}$

(2 marks)

e) Solve the following using matrices:

$$4x_1 + 3x_2 = 4$$
$$-2x_1 - x_2 = 0$$

(2 marks)

f) Differentiate the following:

$$y = \frac{x+1}{\sqrt{x}}$$ **(2 marks)**

g) Find the following integrals:

i) $\int (24 + 7.2x)\,dx$ **(1 mark)**

ii) $\int 0.5x^{-0.5}\ dx$ **(2 marks)**

iii) $\int (48x - 0.4x^{-1.4})\ dx$ **(2 marks)**

h) Solve the following linear programming problem by graphical method:

$$\textit{Minimize}\ \ z = 20x_1 + 40x_2$$
$$\textit{subject to constraints}:$$
$$36x_1 + 6x_2 \geq 108$$
$$3x_1 + 12x_2 \geq 36$$
$$20x_1 + 10x_2 \geq 100$$
$$x_1, x_2 \geq 0$$

(4 marks)

I. Differentiate the following variables:

i) $Y = 3x^{1/2}$ **(2 marks)**

ii) $Y = 1/x^3$ **(2 marks)**

iii) $Y = x^2\, e^{2x}$ **(3 marks)**

II. Find the differential coefficient of

$Y = 7\sin 2x - 3\cos 4x$ **(2 marks)**

QUESTION 2

a) Differentiate

i) $Y = (3x+2)\ (5x^2 - 1 + 2x)$ **(3 marks)**

ii) $Y = \dfrac{2x^2 + 3}{x}$ **(3 marks)**

b) Differentiate the following with respect to the variable

$\dfrac{\sin x}{x}$ **(4 marks)**

c) Find the following integrals:

i) Determine $\int 5e^{3x}$ **(2 marks)**

ii) Determine $\int \frac{2}{5}x^2\ dx$ **(2 marks)**

d) Find the matrix A^{-1} for $A = \begin{bmatrix} 1 & 4 \\ -1 & -3 \end{bmatrix}$ **(2 marks)**

e) The value of a particular asset is estimated by the function $v = 240{,}000e^{-0.04t}$ where V is the value of the asset and t is the age of the asset, measured in years.

i) What is the value of the asset expected to equal to when 4 years old. **(1 mark)**

ii) Determine the general expressions for the instantaneous rate of change in the value of the asset. **(2 marks)**

iii) What is the rate of change expected to equal to when the asset is 10 years old. **(1 mark)**

QUESTION 3

a) Differentiate the following with respect to the variable:

i) $Y = \dfrac{3x^2 - 5}{1 - x^3}$ **(4 marks)**

ii) $Y = \left(\dfrac{3x}{1 - x^2}\right)^5$ **(5 marks)**

b) If a firm spends £650 on fixed costs, what is the total cost function if its marginal cost function is $MC = 82 - 16q + 1.8q^2$ **(2 marks)**

c) Solve the following linear programming problem by graphical method.

$$\begin{aligned} &\textit{Minimize } Z = 5x_1 + 3x_2 \\ &\textit{subject to constraints}: 2x_1 + x_2 \le 1000 \\ &\qquad x_1 \le 400 \\ &\qquad x_2 \le 700 \\ &\qquad x_1, x_2 \ge 0 \end{aligned}$$

(4 marks)

d) Differentiate the following with respect to the variable:

i) $Y = (x+5)(x^2+3)$ **(3 marks)**

ii) $Y = (3x^2 - 5x + 8)^{10}$ **(2 marks)**

QUESTION 4

a) Solve for x and Y by use of Cramer's rule

i) $\begin{aligned} 5x + 3y &= 1 \\ 2x - 3y &= -8 \end{aligned}$

ii) $\begin{aligned} 24x + 2y &= 86 \\ 15x + y &= 52 \end{aligned}$

(4 marks)

b) Differentiate the following functions:

i) $x^2 \, Ln\, x$ **(3 marks)**

ii) $\dfrac{2\cos 3x}{x^3}$ **(4 marks)**

c) Integrate the following functions: **(3 marks)**

$$\int (x^2 - 1)^2 \; dx$$

d) An epidemic is spreading through Cape Town in South Africa. Doctors estimate that the number of persons who will be afflicted by the disease is a function of time since the disease was first detected. The function is $n = f(t) = 300t^3 - 20t^2$ where n equals the number of persons and $0 \le t \le 60$ measured in days.

i) How many people are expected to have caught the disease.

  i. After 10 days **(2 marks)**

  ii. After 30 days **(2 marks)**

ii) What is the instantaneous rate at which the disease is expected to be spreading at *t = 20.*

(2 marks)

QUESTION 5

a) Differentiate the following functions:

i) $Y = 3\sin(4t + 0.12) - 2\cos(3t - 0.72)$

ii) $Y = 5t \sin 2t$

b) Integrate the following

i) $\int x^2 \, dx$

ii) $\int \sqrt{x} \, dx$ **(3 marks)**

c) A dietitian is planning the menu for an elementary school. He plans to serve three main items, all having different nutritional content. He is interest in providing at least the minimum daily requirement of each of the three vitamins in this one meal. The table below summarizes the vitamin content per ounce of each type of food, the cost per ounce, and the minimum daily requirement for each vitamin. Any combination of the three foods may be selected as long as the total serving size is at least 6.0 ounces.

Food	Vitamin			
	1	2	3	Cost per ounce $
1	20 mg	20 mg	20 mg	0.15
2	40 mg	25 mg	30 mg	0.18
3	30 mg	15 mg	25 mg	0.22
Minimum daily requirement	240mg	120 mg	180 mg	

Required:

Formulate the linear programming problem which when solved would determine the number of ounces of each type of food to serve. The objective is to minimize the cost of the meal while satisfying minimum daily requirement levels of the three vitamins as well as the restriction on the minimum serving size. **(4 marks)**

d) A leading processor of sugar has two plants which supply FOUR warehouses. The table summarises weekly capacities of each plant, weekly requirements at each warehouse and shipping cost per tonne (in dollars) between any plant and any warehouse. If x ij equals the number of gallons (in thousands) shipped from plant *i* to depot *j*, formulate the linear programming model which allows for determining the minimum cost allocation schedule. Plant capacities are not to be violated, and deport demands are to be satisfied by the schedule.

	Depot				
Plant	1	2	3	4	Supply 1,000 gal
	50	40	35	20	1,000
	30	45	40	60	1,400
	60	25	50	30	1,800
Demand, 1,000 gal	800	750	650	900	

(4 marks)