TECHNICAL UNIVERSITY OF MOMBASA

A Centre of Excellence

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATION FOR THE SECOND SEMESTER IN THE FOURTH YEAR OF BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

MAY 2016 SERIES EXAMINATION

UNIT CODE: AMA 4426

UNIT TITLE: STOCHASTIC PROCESSES

TIME ALLOWED: 2HOURS

INSTRUCTIONTO CANDIDATES:

You should have the following for this examination

- Mathematical tables
- Scientific Calculator

This paper consists of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

QUESTION ONE (30 MARKS)

(a) (i) Define stationarity in the strict sense and stationarity in the weak sense. (4 marks)

(ii) Show that a stochastic process with probability generating function given by :

$P(S) = e^{\lambda t(s-1)}$ is non stationary in the weak sense.

(4 marks)

(b) Let $\{X_n : n = 1,2,3, \dots \dots\}$ be a stochastic process with probability distribution.

$$P(X_n = K) = \begin{cases} pq^{k-1} & , k = 2,3,4, \dots \dots . \\ 0 & elsewhere \end{cases}$$

Find the probability generating function of $\{X_n\}$. Hence obtain the mean and the variance of the process.

(14 marks)

(c) Consider Fibonacci number given by $f_0 = 0 , f_1 = 1$

$$f_n = f_{n-1} + f_{n-2} \qquad , (n \geq 2)$$

Find the generating function of these numbers

(8 marks)

QUESTION TWO (20 MARKS)

A stochastic process with state space $(E_1 E_2 E_3 E_4)$ has the following transition probability matrix

$$P = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Classify the states of this process.

(20 marks)

QUESTION THREE (20 MARKS)

(a) Define the following terms:

(i) Stochastic process (2 marks)

(ii) Bernoulli process (2 marks)

(iii) Markov chain (2 marks)

(b) The joint distribution of two random variables X and Y is given by :

$$P_{jk} = P\{X = j, Y = k\} = \begin{cases} q^{j+k}p^2 \ , j = 0,1,2,3, \dots, k = 0,1,2,3, \dots, p + q = 1 \\ 0 \qquad \qquad \qquad \qquad \qquad \qquad otherwise \end{cases}$$

Obtain the following:

(i) Bivariate p.g.f of X and Y (3 marks)

(ii) Variance of X and Y (10 marks)

(iii) Covariance of X and Y (1 mark)

QUESTION FOUR (20 MARKS)

(a) Let X be a random variable such that $P(X = k) = P_k$

$$P(X > k) = q_k = \sum_{r=k+1}^{\infty} P_r \quad , k > 0$$

If

$$P(S) = \sum_{k=0}^{\infty} P_k S^k \qquad and \quad Q(S) = \sum_{k=0}^{\infty} q_k s^k$$

Show that $(1 - S)Q(S) = 1 - P(S) \quad and \ that \ E(X) = Q(1)$

(10 marks)

(b) Suppose that $X_i \quad i = 1,2$ are two independent random variables with

$$P(X_i = k) = p_i q_i^k \quad i = 1,2 \quad and \ k = 0,1,2, \dots \dots ..$$

Find the bivariate p.g.f $P(S_1,S_2)$ of the pair (X_1,X_2) and from the form of the p.g.f , the sum $S_1=X_1+X_2$.

Verify that

$$P(S_2 = k) = \sum_{r=0}^{k} q_1^r p_1 q_2^{k-r} p_2$$

(10 marks)

QUESTION FIVE (20 MARKS)

(a) Solve the differential difference equation

$$U_n'(t) = U_{n-1}{}^{(t)} \quad t \geq 0 \quad , n = 1,2,3, \dots \dots$$

Given the initial conditions:

$U_n(t) = 1\ , t \geq 0\ \ and\ \ U_n(0) = \ 0\ \ , n \neq 0$

(10 marks)

b) The probability function

$P_{n=}\ P(N = n)$ n=0, 1,2,3,…………….. of a random variable N satisfy the difference equation

$P_{n+1} - \ (1 + a)P_n + \ aP_{n-1} = 0 \quad , n \geq 1 \quad and\ - P_1 + aP_0 = 0 \quad , 0 < a < 1$

Solve the equation using the method characteristic function. (10 marks)