TECHNICAL UNIVERSITY OF MOMBASA
BFSQ/BTAC **A**

AMA4109: CALCULUS FOR SCIENCES

INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO

QUESTION ONE (30MARKS)

(a) If $A = \{x \mid -3 \le x \le 2, x \in \mathbb{Z}\}$ and a function $f : A \to \mathbb{Z}$ is defined by $f(x) = x^2 \ \forall \ x \in A$, find the range of f and state whether it is onto or not. (3mks)

(b) Find the derivatives of the following functions with respect to x

(i) $y = x^5 \sin x$. (2mks)

(ii) $x^3 + 3y^6 = y^3$. (3mks)

(c) Evaluate the following limits

(i) $\lim_{x \to -1} \frac{x+1}{x^2 - 1}$. (3mks)

(ii) $\lim_{x \to \infty} \sqrt{x^2 + 1} - x$. (3mks)

(d) (i) Use first principles to differentiate $f(x) = x^2 + 2$. (3mks)

(ii) Hence use the result in (i) to find the tangent line to $f(x)$ at $x = -2$. (3mks)

(e) Find the following integrals

(i) $\int 20x(x^2+3)^7 \, dx$. (3mks)

(ii) $\int xe^{-x^2} \, dx$. (3mks)

(f) Given that $x = t^3 - 3t^2$ and $y = t^3 - 3t$, find $\frac{dy}{dx}$. (4mks)

QUESTION TWO (20MKS)

(a) Suppose $f(2) = 11$, $f'(2) = 12$, $g(2) = 7$ and $g'(2) = 4$ .Evaluate

$$\left(\frac{f}{g}\right)'(2) + \left(fg\right)'(2).$$ (3mks)

(b) (i) Let *f* be a function defined at points near a (except possibly at a). Let *L* be a real number. Use $\varepsilon - \delta$ notation to define *L* as a limit of *f*. (2mks)

(ii) Use the definition in (i) to show that

$\lim_{x\to 2} 2x-3=1$. (4mks)

(c) Let $f, g : \mathcal{R} \to \mathcal{R}$ such that $f(x) = x+1$ and $g(x) = x^2+3$.

(i) Compute $h = f_o g$. (2mks)

(ii) Find f^{-1}, g^{-1} and h^{-1}. (3mks)

(iii) Compute $g^{-1}{}_o f^{-1}$, compare this to (ii) and make any relevant deduction.

(2mks)

(d) Show that if $y = \sec^{-1}\sqrt{1+x^2}$ then $\frac{dy}{dx} = \frac{1}{1+x^2}$. (4mks)

QUESTION THREE (20MKS)

(a) Suppose $g(x) = f^{-1}(x)$ and $G(x) = \frac{1}{g(x)}$.Given that $f(3) = 2$, and $f^{-1}(3) = \frac{1}{9}$, find $G^{/}(2)$. (4mks)

(b) Calculate the volume of the solid generated by rotating about the *x*-axis the area bounded by $f(x) = 4 - x^2$ and the *x*-axis. (5mks)

(c) Find the linearization of $f(x) = \sqrt{x+3}$ at $x = 1$ and use it to approximate $\sqrt{4.05}$.

(5mks)

(d) The parametric equations of a curve are $x = e^t$ and $y = \sin t$ .Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ .Hence show that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$. (6mks)

QUESTION FOUR (20MKS)

(a) Find the equation of the normal to the curve $y = x + \sqrt{x}$ at $(1, 2)$. (3mks)

(b) Find the absolute maximum and minimum of $f(x) = x^3 - 12x + 1$ on $-3 \le x \le 5)$.

(4mks)

(c) Decompose the following rational fraction

$$\frac{2x^2+6x-4}{x(x+2)^4}$$ (4mks)

(d)Air is being pumped into a spherical balloon so that its volume increases at 100cm^3/s. How fast is the radius increasing when the diameter is 50 cm? (3mks)

(e) A particle P travels in a straight line and its distance x meters from a fixed point A on the line at time t seconds is given by $x = 2t^3 - 15t^2 + 36t + 20$. Find the values of x at the points where the velocity is zero. (6mks)

QUESTION FIVE(20MKS)

(a) Use logarithmic differentiation to find $\frac{dy}{dx}$

(i) $y = x^2\sqrt{(x+2)}$. (2mks)

(ii) $y = \log_2(x^3 + 5)$ (4mks)

(b) Integrate

(i) $\int \frac{x^2}{\cos^2 x^3}\, dx$ (3mks)

(ii) $\int \theta^2 \sin\theta \, d\theta$ (3mks)

(c) A metal sheet has measurements 8 by 5 metres. Equal squares of side x metres are removed from each corner and the edges are then turned up to make an open box of volume V m^3.

(i) Show that $V = 40x - 26x^2 + 4x^3$. (2mks)

(ii) Find the maximum possible volume and the corresponding value of x. (6mks)