TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

AMA 4410: PARTIAL DIFFERENTIAL EQUATIONS 1

END OF SEMESTER EXAMINATION

SERIES:APRIL2016

TIME:2HOURS

DATE: Pick DateMay2016

Instructions to Candidates

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

This paper consists of **FIVE** questions. Attempt question ONE (Compulsory) and any other TWO questions.

Do not write on the question paper. **PAPER 2**

QUESTION ONE (30 MARKS)

a. Solve the linear PDE $p+3q=5z+\tan(y-3x)$ (5 marks)

b. Derive a PDE by eliminating the arbitrary function ϕ from the equation $\phi\left(x^3+y^3+z^3, z^3-2x^2y^2\right)=0$ (6 marks)

c. Classify each of the following equations as elliptic, parabolic or hyperbolic

   i. $u_{xx}+u_{yy}=0$ (2marks)

   ii. $u_{xx}+3u_{xy}+4u_{yy}+5u_x-2u_y+4u=2x-3y$ (2marks)

d. Find the general solution of $r-3s+2t=e^{x+y}$ [7 Marks]

e. Find the equation of the surface satisfying the equation $4yzp+q+2y=0$ and passing through $y^2+z^2=1, x+z=2$. [8 marks]

QUESTION TWO (20 MARKS)

a. Find the complete integral of $2p_1x_1x_3 + p_2^2p_3 + 3p_2x_3^2 = 0$ using the Jacobi's method. (10 marks)

b. Use Charpit's method to find the complete integral of $xp + q = p^2$ (10 marks)

QUESTION THREE (20 MARKS)

a. Derive a PDE by eliminating the arbitrary constants a and b from
$z = ax^2 + by^2 + ab$. (5 marks)

b. A string of length L is stretched between points $(0,0)$ and $(L,0)$ on the x axis. At time $t = 0$ it has a shape given by $f(x), \quad 0 \le x \le L$ and it is released from rest. Find the displacement of the string at any latter time. (15 marks)

QUESTION FOUR (20 MARKS)

a. Solve the heat conduction equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t}$, k =constant subject to the following boundary conditions:
$$\begin{cases} u(x,0) = f(x), & 0 \le x \le L \\ \frac{\partial u}{\partial x}\Big|_{x=0} = \frac{\partial u}{\partial x}\Big|_{x=L} = 0, & t \ge 0 \end{cases}$$
[12 Marks]

b. Solve $(D_x^2 - D_xD_y - 2D_y^2 + 3D_x + 2)z = 0$ [8 Marks]

QUESTION FIVE (20 MARKS)

a. Show that the orthogonal trajectories on the hyperboloid $x^2 + y^2 - z^2 = 1$ of a conic in which it is cut by the system of planes $x + y = c$ are the curves of intersection with the family of surfaces $(x - y)z = k$ where k is a parameter. (13marks)

Find the integral curves of the equations $\dfrac{dx}{x^2 - y^2 - z^2} = \dfrac{dy}{2xy} = \dfrac{dz}{2xz}$ (7 marks)