TECHNICAL UNIVERSITY OF MOMBASA

## FACULTY OF APPLIED AND HEALTH SCIENCE

## DEPARTMENT OF MATHEMATICS AND PHYSICS

## UNIVERSITY EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE. <br> AMA 4326: FLUID MECHANICS II

## END OF SEMESTER EXAMINATION: MAY 2016 - SERIES

TIME: 2 HOURS

## Instructions to Candidates

You should have the following to do this examination:
-Answer Booklet, examination pass and student ID
Do not write on the question paper.
Answer question One and any other two

## Question one: 30 marks (Compulsory)

a) Define the following terms.
i) Circulation.
ii) Stream function.
2 mark
b) Find a relevant stream function to a set of velocity components for a steady incompressible flow if $U=x+y$ and $V=x-y$.

4 marks
c) Show that the discharge per unit width between two parallel plates distance $b$ apart when one plate is moving at velocity U while the other one is held stationary for the condition of zero shear stress at the fixed plate is $q=\frac{U b}{3}$.

6 marks
d) Given that $U=-4 a x\left(x^{2}-3 y^{2}\right)$ and $V=4 a y\left(3 x^{2}-y^{2}\right)$ examine whether these velocity components represent a physically possible 2 dimensional flow, if so is the flow rotational or irrotational.
e) Obtain the velocity profile of a plane parallel (coutte) flow between two plates moving at a constant velocity $\mathrm{U}_{0}$ but in opposite direction, the distance y is measured from a fixed point 0 and the distance between the plates is 8 mm if $\frac{\partial p}{\partial x} \neq 0$; determine the skin friction on each of the plates.
f) Discuss the flow whose complex potential function is given by $w=z^{2}$.

## Question TWO (20 marks)

a) The velocity potential for a two dimensional flow is $\phi=x(2 y-1)$ at the point $\mathrm{P}(4,5)$ determine : i) the velocity. 4 marks
ii) The value of the stream function. 4 marks
b) Discuss the complex velocity potential of an incline flow to the x axis at an angle $\alpha^{0}$ if $\nabla \phi=-\vec{q}$.

7 marks
c) A fluid is flowing steadily between two fixed parallel plates under constant pressure gradient $P_{0}=\frac{-\partial P}{\partial x}$. Show that the velocity distribution of this flow is $U=\frac{P_{0}}{2 \mu}\left(d^{2}-y^{2}\right)$ where 2 d is the distance between the 2 plates and y is measured from the midpoint.

5 marks

## Question THREE (20 marks)

a) Write the complete Navier Stokes equation for conservation of momentum then name all the terms in the equation.
b) The velocity components for a fluid flow are $U=a+b y-c z, V=d-b x-e z$ and $W=f+c x-e y$, where $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \mathrm{e}$ and f are arbitrary constants.
$\begin{array}{llc}\text { i) } & \text { Show that it is a possible case of fluid flow. } & 2 \text { marks } \\ \text { ii) } & \text { Is the fluid flow irrotational. } & 3 \text { marks } \\ \text { iii) } & \text { If not determine the vorticity of the flow. } & 2 \text { marks }\end{array}$
c) Suppose that a liquid is in the region of the positive side of $x$ - axis with rigid boundary $y=0$ and there is a source of strength $-m$ at $(0, a)$ and an equal sink at $(0, b)$. If the pressure on the negative side of the boundary is the same as the pressure at infinity, show
that if the fluid satisfies the no - slip condition on the boundary then the resultant pressure on the boundary is given by $P=\frac{1}{2} \rho \int_{-\infty}^{+\infty} \frac{4 m^{2} x^{2}\left(b^{2}-a^{2}\right)}{\left(x^{2}+a^{2}\right)^{2}\left(x^{2}+b^{2}\right)^{2}} d x$.

10 marks

## Question FOUR (20 marks)

a) Find the equation of the streamlines due to 2 line sinks each of strength $m$ through the point $(-\mathrm{C}, 0)$ and $(\mathrm{C}, 0)$ with a uniform line source of strength 2 m through the origin.

9 marks
b) In a 2 dimensional flow the velocity function is given by the expression $\phi=x^{2}-y^{2}$ :
i) Determine velocity components in x and y directions.

2 marks
ii) Show that the velocity components satisfy the conditions of continuity flow and irrotationality.

4 marks
iii) Determine stream function and the flow rate between the streamlines $(2,0)$ and $(2,2)$

3 marks
iv) Show that streamlines and potential lines intersect orthogonally at point $(2,2)$. 2 marks

## Question FIVE (20 marks)

a) In a 2 dimensional flow the velocity components are $\mathrm{U}=\mathrm{Cy}$ where C is a constant and $\mathrm{V}=0$, find the circulation about the circle $x^{2}+y^{2}-2 \boldsymbol{a} y=0$ situated in the flow if $\boldsymbol{a}$ is the radius of the circle. 5 marks
b) The radial velocity profile in a pipe is given by $U=U_{\max }\left(\frac{1-r}{R}\right)^{n}$ where U is the velocity at a radial distance $r, U_{\text {max }}$ is the maximum velocity and $R$ is the radius of the pipe. Derive an equation for the average velocity in the pipe.

8 marks
c) Discuss the flow equation of a line vortex whose complex velocity potential is $w=\frac{i k \log z}{2 \pi}$

## THE END

