

- e) Obtain the velocity profile of a plane parallel (couette) flow between two plates moving at a constant velocity U_0 but in opposite direction, the distance y is measured from a fixed point 0 and the distance between the plates is 8mm if $\frac{\partial p}{\partial x} \neq 0$; determine the skin friction on each of the plates. 7 marks
- f) Discuss the flow whose complex potential function is given by $w = z^2$. 4 marks

Question TWO (20 marks)

- a) The velocity potential for a two dimensional flow is $\phi = x(2y - 1)$ at the point P(4,5) determine : i) the velocity. 4 marks
 ii) The value of the stream function. 4 marks
- b) Discuss the complex velocity potential of an incline flow to the x axis at an angle α^0 if $\nabla \phi = -\vec{q}$. 7 marks
- c) A fluid is flowing steadily between two fixed parallel plates under constant pressure gradient $P_0 = \frac{-\partial P}{\partial x}$. Show that the velocity distribution of this flow is $U = \frac{P_0}{2\mu}(d^2 - y^2)$ where $2d$ is the distance between the 2 plates and y is measured from the midpoint. 5 marks

Question THREE (20 marks)

- a) Write the complete Navier Stokes equation for conservation of momentum then name all the terms in the equation. 3 marks
- b) The velocity components for a fluid flow are $U=a+by-cz$, $V=d-bx-ez$ and $W=f+cx-ey$, where a, b, c, d, e and f are arbitrary constants.
- i) Show that it is a possible case of fluid flow. 2 marks
- ii) Is the fluid flow irrotational. 3 marks
- iii) If not determine the vorticity of the flow. 2 marks
- c) Suppose that a liquid is in the region of the positive side of x- axis with rigid boundary $y=0$ and there is a source of strength $-m$ at $(0, a)$ and an equal sink at $(0, b)$. If the pressure on the negative side of the boundary is the same as the pressure at infinity, show

that if the fluid satisfies the no – slip condition on the boundary then the resultant

pressure on the boundary is given by
$$P = \frac{1}{2} \rho \int_{-\infty}^{+\infty} \frac{4m^2 x^2 (b^2 - a^2)}{(x^2 + a^2)^2 (x^2 + b^2)^2} dx .$$

10 marks

Question FOUR (20 marks)

- a) Find the equation of the streamlines due to 2 line sinks each of strength m through the point $(-C, 0)$ and $(C, 0)$ with a uniform line source of strength $2m$ through the origin.

9 marks

- b) In a 2 dimensional flow the velocity function is given by the expression $\phi = x^2 - y^2$:

- i) Determine velocity components in x and y directions. 2 marks

- ii) Show that the velocity components satisfy the conditions of continuity flow and irrotationality. 4 marks

- iii) Determine stream function and the flow rate between the streamlines $(2,0)$ and $(2,2)$ 3 marks

- iv) Show that streamlines and potential lines intersect orthogonally at point $(2, 2)$. 2 marks

Question FIVE (20 marks)

- a) In a 2 dimensional flow the velocity components are $U=Cy$ where C is a constant and $V=0$, find the circulation about the circle $x^2 + y^2 - 2a y=0$ situated in the flow if a is the radius of the circle. 5 marks

- b) The radial velocity profile in a pipe is given by $U = U_{\max} \left(\frac{1-r}{R} \right)^n$ where U is the velocity at a radial distance r , U_{\max} is the maximum velocity and R is the radius of the pipe. Derive an equation for the average velocity in the pipe. 8 marks

- c) Discuss the flow equation of a line vortex whose complex velocity potential is $w = \frac{ik \log z}{2\pi}$

7 marks

THE END