



# TECHNICAL UNIVERSITY OF MOMBASA

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*Faculty of applied and Health Sciences*

DEPARTMENT OF MATHEMATICS AND PHYSICS

**UNIVERSITY EXAMINATION FOR:**

**BACHELOR OF MATHEMATICS AND COMPUTER SCIENCE**

**AMA 4323: ORDINARY DIFFERENTIAL EQUATIONS II**

**END OF SEMESTER EXAMINATION**

**SERIES: MAY 2016**

**TIME: 2 HOURS**

**DATE: 2016**

**PAPER A**

**Instructions to Candidates**

You should have the following for this examination

*-Answer Booklet, examination pass and student ID*

This paper consists of 5 questions. Question one is compulsory. Answer any other two questions

**Do not write on the question paper.**

**QUESTION ONE (COMPULSORY)**

- a) Obtain the general solution to the following homogeneous linear system

$$x_1' = x_2$$

$$x_2' = 3x_2 - 2x_1$$

(5 marks)

- b) Find a solution of the initial Value problem  $\frac{dy}{dx} = x^2$ ,  $x_0 = 2$ ,  $y_0 = 1$  using the uniqueness and

existence theorem

(5 marks)

c) Reduce the fourth order equation to first order systems

$$\frac{d^4 y}{dx^4} - 5 \frac{d^3 y}{dx^3} + 7 \frac{d^2 y}{dx^2} + 9 \frac{dy}{dx} - 6y = e^x \quad (5 \text{ marks})$$

d) Solve  $\frac{d^3 y}{dx^3} = xe^x$  (5 marks)

e) Solve  $x^2 y \frac{d^2 y}{dx^2} + \left( x \frac{dy}{dx} - y^2 \right) = 0$  (5 marks)

f) Locate and classify the singular points of the equation

$$(x^4 - 2x^3 + x^2) \frac{d^2 y}{dx^2} + 2(x-1) \frac{dy}{dx} + x^2 y = 0 \quad (5 \text{ marks})$$

## QUESTION TWO

a) Consider a first order vector equation  $X'(t) = AX(t) + B(t)$  where  $A$  is an  $n \times n$  matrix of real numbers,  $X(t)$ ,  $B(t)$  are column vectors. Obtain the characteristic polynomial, characteristic equation and Eigen values of matrix  $A$ . (12 marks)

b) Solve  $zydx = zxdy + y^2 dz$  (8 marks)

## QUESTION THREE

a) Find the two independent series solutions of the Bessel's equation  $x^2 y'' + xy' + (x^2 - 1)y = 0$  (14 marks)

b) Solve the first order system  $\frac{dx}{dt} = y$ ,  $\frac{dy}{dt} = -2x + 3y$  (6 marks)

#### QUESTION FOUR

a) Solve the system  $X' = AX$  where  $A = \begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & 3 \\ 0 & 3 & 1 \end{pmatrix}$  (13 marks)

b) Obtain the roots of the indicial equation of  $9x^2 y'' + (x+2)y = 0$  (7 marks)

#### QUESTION FIVE

a) Solve the system of linear equations

$$\begin{aligned} x'(t) &= 3x(t) - 4y(t) \\ y'(t) &= 4x(t) - 7y(t) \end{aligned} \quad (12 \text{ marks})$$

b) Show the convergence of the initial value problem

$$\frac{dy}{dx} = y; \quad x_0 = 0, \quad y_0 = 1 \quad (8 \text{ marks})$$