## BACHELOR OF MATHEMATICS AND COMPUTER SCIENCE

## AMA 4305: OERATIONS RESEARCH 1

END OF SEMESTER EXAMINATION
SERIES:APRIL2016

## TIME:2HOURS

DATE:Pick DateMay2016

## Instructions to Candidates

You should have the following for this examination
-Answer Booklet, examination pass and student ID
This paper consists of FIVE questions. Attempt question ONE (Compulsory) and any other TWO questions.
Do not write on the question paper.
PAPER 2
QUESTION ONE (30 MARKS)
a) Define the following terms

| i. | Linear programming | [1 Mark] |
| ---: | :--- | :--- |
| ii. | Objective function | [1 Mark] |
| iii. | A dummy activity | [1 Mark] |

a) Describe decision making under certainty.
b) A management is faced with the problem of choosing one of the products for manufacturing. The probability matrix after market research for the two products was as follows:-

|  | State of Nature |  |  |
| :---: | :---: | :---: | :---: |
| Act | Good | Fair | Poor |
| Product A | 0.75 | 0.15 | 0.1 |
| Product B | 0.6 | 0.3 | 0.1 |

The profits that the management can make for different levels of market acceptability of the products are as follows:-

|  | State of Nature <br>  <br> (Profit (in Ksh) if the market is) |  |  |
| :---: | :---: | :---: | :---: |
| Act | Good | Fair | Poor |
| Product A | 35,000 | 15,000 | 5,000 |
| Product B | 50,000 | 20.000 | Loss of 3,00 |

Calculate the expected value of the choice of alternatives and advise the management.[6Mks]
c) Construct a network for the project whose activities and their precedence relationships are as given below

| Activity | A | B | C | D | E | F | G | H | I |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Predecessor | - | A | A | - | D | C,E | F | D | G,H |

d) Use simplex method to solve the following linear programming problem:

Maximize $Z=15 x_{1}+20 x_{2}$ subject to the constraints

$$
\begin{align*}
& x_{1}+2 x_{2} \leq 160 \\
& 2 x_{1}+x_{2} \leq 180  \tag{8Marks}\\
& x_{1}, x_{2} \geq 0
\end{align*}
$$

e) Solve the transportation problem with the cost matrix given below using the North West Corner Rule.

| Destination <br> Origin | $\mathbf{D}_{\mathbf{1}}$ | $\mathbf{D}_{\mathbf{2}}$ | $\mathbf{D}_{\mathbf{3}}$ | $\mathbf{D}_{\mathbf{4}}$ | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{O}_{\mathbf{1}}$ | 1 | 5 | 3 | 3 | $\mathbf{3 4}$ |
| $\mathbf{O}_{\mathbf{2}}$ | 3 | 3 | 1 | 2 | $\mathbf{1 5}$ |
| $\mathbf{O}_{\mathbf{3}}$ | 0 | 2 | 2 | 3 | $\mathbf{1 2}$ |
| $\mathbf{O}_{\mathbf{4}}$ | 2 | 7 | 2 | 4 | $\mathbf{1 9}$ |
| Required | $\mathbf{2 1}$ | $\mathbf{2 5}$ | $\mathbf{1 7}$ | $\mathbf{1 7}$ |  |

## QUESTION TWO (20 MARKS)

a) What do you understand by transportation model?
b) Obtain the initial solution for the following Transportation Problem using
i. Least cost method
ii. VAM

| Destination <br> Origin | $\mathbf{D}_{\mathbf{1}}$ | $\mathbf{D}_{\mathbf{2}}$ | $\mathbf{D}_{\mathbf{3}}$ | Supply |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{O}_{\mathbf{1}}$ | 6 | 4 | 1 | $\mathbf{1 4}$ |
| $\mathbf{O}_{\mathbf{2}}$ | 8 | 9 | 2 | $\mathbf{1 6}$ |
| $\mathbf{O}_{\mathbf{3}}$ | 4 | 3 | 6 | $\mathbf{5}$ |
| Required | $\mathbf{6}$ | $\mathbf{1 0}$ | $\mathbf{1 5}$ |  |

f) Convert the following LPP into a maximization problem:

Minimize $z=3 x_{1}+2 x_{2}+x_{3} \quad$ subject to

$$
\begin{align*}
& 3 x_{1}+2 x_{2}-x_{3} \geq 3 \\
& 3 x_{1}-3 x_{2}+x_{3} \leq-6 \\
& 2 x_{1}+x_{2}+4 x_{3} \geq 18 \\
& x_{1}, x_{2}, x_{3} \geq 0 \tag{5Marks}
\end{align*}
$$

## QUESTION THREE (20 MARKS)

The following table shows the jobs of a network along with their time estimates:

| Activity | Estimated duration (weeks) |  |  |
| :---: | :---: | :---: | :---: |
|  | Optimistic | Most likely | Pessimistic |
| $1-2$ | 1 | 7 | 13 |
| $1-6$ | 2 | 5 | 14 |
| $2-3$ | 2 | 14 | 26 |
| $2-4$ | 2 | 5 | 8 |
| $3-5$ | 7 | 10 | 19 |
| $4-5$ | 5 | 5 | 17 |
| $6-7$ | 5 | 8 | 29 |
| $5-8$ | 3 | 3 | 9 |
| $7-8$ | 8 | 17 | 32 |

a) Find the expected duration and variance of each activity
b) Draw the project network
c) Find the critical path
d) Calculate the expected project length
e) Find the probability that the project is completed in 40 days.

## QUESTION FOUR (20 MARKS)

a) A person requires 10,12 and 12 units of chemicals $\mathrm{A}, \mathrm{B}$ and C respectively for his garden. A liquid product contains 5, 2 and 1 units of $\mathrm{A}, \mathrm{B}$ and C respectively per jar. A dry product contains 1,2 and 4 units of A, B and C per carton. If the liquid product sells for $\$ 3$ per jar and the dry product sells for $\$ 2$ per carton, find by graphical method, the number of units of each package that should be purchased in order to minimize the cost and meet the requirements?
b) A company purchases 9000 parts of a machine for its annual requirements, ordering one month's usage at a time. Each part costs $£ 20$. The ordering cost per order is $£ 15$ and the carrying charges are $15 \%$ of the average inventory per year. You have been asked to suggest a more economical purchasing policy for the company. What advice would you offer and how much would it save the company per year?
[10 Marks]

## QUESTION FIVE (20 MARKS)

a) What do you understand by simulation?
[2 Marks]
b) A tourist car operator finds that during the past few months, the car's use has varied so much that the cost of maintaining it varied considerably. During the past 200 days, the demand for the car fluctuated as follows:

| Trips per week | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Frequency | 16 | 24 | 30 | 60 | 40 | 30 |

Using the following random numbers, simulate the demand for a 10 week period.
$\begin{array}{llllllllll}82 & 96 & 18 & 96 & 20 & 84 & 56 & 11 & 52 & 03\end{array}$
[6 Marks]
c) A small ink manufacturer produces a certain type of ink at a total average cost of Ksh3 per small bottle and sells it at a price of Ksh5 per bottle. The ink is produced over the weekend and is sold during the following week. According to past experience, the weekly demand has never been less than 78 or greater than 80 bottles in his place. Formulate the pay off table

