

TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHELOR OF SCIENCE IN; STATISTICS AND COMPUTER

SCIENCE,

MATHEMATICS AND COMPUTER SCIENCE

AMA 4212: VECTOR ANALYSIS

END OF SEMESTER EXAMINATION

SERIES: APRIL 2016

TIME: 2HOURS

DATE: Pick Date Select Month Pick Year

Instructions to Candidates

You should have the following for this examination

-Answer Booklet, examination pass and student ID

This paper consists of Choose No questions. Attempt Choose instruction.

Do not write on the question paper.

QUESTION ONE (30 MARKS)

- a) Explain each of the following, giving an example of each.
- i) A vector (2marks)
 - ii) A scalar (2marks)
- b) Show that addition of vectors is commutative. (4marks)
- c) Find the projection of the vector $\hat{i} - 2\hat{j} + \hat{k}$ on $4\hat{i} - 4\hat{j} + 7\hat{k}$. (3marks)
- d) Find the divergence and curl of the vector

$$\vec{V} = (xyz)\hat{i} + (3x2y)\hat{j} + (xz^2 - y^2z)\hat{k} \text{ at the point } (2,-1,1) \quad (6\text{marks})$$

e) A vector field is given by $\vec{A} = (x^2 + xy^2)\hat{i} + (y^2 + x^2y)\hat{j}$ show that the field is irrotational. (4marks)

f) Given $\vec{a} = \hat{i} + \hat{j} - \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. Find the vector $\vec{a} \times (\vec{b} \times \vec{c})$ (5marks)

g) State Green's theorem. (4marks)

QUESTION TWO (20MARKS)

a) Find the angle between the vectors $(2\hat{i} + 6\hat{j} + 3\hat{k})$ and $(12\hat{i} - 4\hat{j} + 3\hat{k})$. (5marks)

b) Use Greens' theorem in a plane to evaluate the integral $\oint_c (2x^2 - y^2)dx + (x^2 + y^2)dy$

Where c is the boundary in the xy plane of the area enclosed by the x-axis and the semi-circle $x^2 + y^2 = 1$ in the upper half xy plane. (9marks)

c) A force $\vec{F} = 3\hat{i} + 2\hat{j} - 4\hat{k}$ is applied at the point $(1,-1,2)$. Find the moment of the force about the point $(2,-1,3)$ (6marks)

QUESTION THREE (20MARKS)

a). Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ taken round the rectangle bounded by the lines $x = \pm a, y = 0, y = b$. (15marks)

b). Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point $p(1,2,3)$ in the direction of the line PQ where Q is the point $(5,0,4)$ (5marks)

QUESTION FOUR (20MARKS)

a). Evaluate:

i) $\operatorname{div}\left[\frac{\vec{r}}{r^3}\right]$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. (7marks)

ii) $\int_c \vec{F} \cdot d\vec{r}$ where c is the arc of the parabola $y = 2x^2$ from $(0,0)$ to $(1,2)$ and $\vec{F} = 3xy\hat{i} - y^2\hat{j}$ (7 marks)

b). Constant forces $\vec{P} = 2\hat{i} - 5\hat{j} + 6\hat{k}$ and $\vec{Q} = -\hat{i} + 2\hat{j} - \hat{k}$ act on a particle. Determine the work done when the particle is displaced from $A(4,-3,-2)$ to $B(6,1,-3)$

(6marks)

QUESTION FIVE (20 MARKS)

a) If $\vec{a} \times \vec{r} = \vec{b} + \lambda\vec{a}$ and $\vec{a} \cdot \vec{r} = 3$ where $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = -\hat{i} - 2\hat{j} + \hat{k}$. Find \vec{r} and λ

(9marks)

b) Find the area of a parallelogram whose adjacent sides are $\hat{i} - 2\hat{j} + 3\hat{k}$ and $2\hat{i} + \hat{j} - 4\hat{k}$

(4marks)

c) Show that the four points $(3,-2,4)$, $B(6,3,1)$, $C(5,7,3)$ and $D(2,2,6)$ are coplanar. (7marks)