TECHNICAL UNIVERSITY OF MOMBASA

SMA 2370 CALCULUS IV 2015/2016

END OF SEMESTER TWO YEAR THREE EXAMINATION FOR THE DEGREE OF BACHELOR OF CIVIL AND MECHANICAL ENGINEERING

QUESTION ONE (30MARKS) COMPULSORY

a) If $\phi = x^2 yz^3$ and $A = xzi - y^2 j + 2x^2 yk$, find

i. $\nabla\phi$ (3mks)

ii. $Div(\phi A)$ (3mks)

iii. $Curl(\phi A)$ (3mks)

b)

i. Find the directional derivative $U = 2x^3 y - 3y^2 z$ at $P(1,2,-1)$ in a direction toward $Q(3,-1,5)$ (6mks)

ii. In what direction from P is the directional derivative maximum? (1mk)

iii. What is the magnitude of the maximum directional derivative (2mks)

c) Identify the surface generated by the equation $r^2 - 4r\cos\vartheta = 14$ (4mks)

d) Sketch the region R in the $xy-$plane bounded by $y = x^2, x = 2, y = 1$ (2mks)

e) Evaluate the double integral $\iint (x^2 + y^2)dxdy$ (3mks)

f) Evaluate $\int_{(0,1)}^{(1,2)} (x^2 + y^2)dxdy$ (3mks)

QUESTION TWO (20MARKS)

a) Evaluate $\int_c A.dr$ from $(0,0,0)$ to $(1,1,1)$ along the following paths if

$A = (3x^2 - 6yz^2)i + (2y + 3xz)j + (1 - 4xyz^2k)$

i. $x = t, y = t^2, z = t^3$ (5mks)

ii. The straight lines from $(0,0,0)$ to $(0,0,1)$ then to $(1,1,1)$ (12mks)

iii. The straight line joining $(0,0,0)$ and $(1,1,1)$ (3mks)

QUESTION THREE (20MARKS)

a) Verify green's theorem in the plane for $\int_c (2xy - x^2)dx + (x + y^2)dy$ where C is the closed curve of the region bounded by $y = x^2$ and $y^2 = x$ (20mks)

QUESTION FOUR (20MARKS)

a) Evaluate $\iint r.nds$ where s is the closed surface. (5mks)

b) Prove that $F = (2xz^3 + 6y)i + (6x - 2yz)j + (3x^2z^2 - y^2)$ is a conservative force field.

c) Find the directional derivative of $F = x^2yz^3$ along the curve $x = e^{-u}, y = 2\sin u + 1, z = u - \cos u$ at the point where $u = 0$ (10mks)

QUESTION FIVE (20MARKS)

a) Given that $f(x, y) = 2x^3 + 3xy^2$ find

  i. f_{xx} (1mk)

  ii. f_{xy} (1mk)

  iii. f_{yy} (1mk)

b) Find the equation for the tangent plane to the surface $x^2yz + 3y^2 = 2xz^2 - 8z$ at the point $(1,2,-1)$ (7mks)

c) Find the shortest distance from the origin to the hyperbola $x^2 + 8xy + 7y^2 = 225, z = 0$ (10mks)