TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF ELECTRICAL & ELECTRONICS

ENGINEERING, BUILDING & CIVIL ENGINEERING AND

MECHANICAL & AUTOMOTIVE ENGINEERING

UNIVERSITY EXAMINATION FOR BACHELOR OF SCIENCE IN CIVIL ENGINEERING, MECHANICAL ENGINEERING & ELECTRICAL ENGINEERING

SMA 2371: PARTIAL DIFFERENTIAL EQUATIONS

END OF SEMESTER EXAMINATION

SERIES:APRIL2016

TIME:2HOURS

DATE: Pick DateMay2016

Instructions to Candidates

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

This paper consists of **FIVE** questions. Attempt question ONE (Compulsory) and any other TWO questions.

Do not write on the question paper. **PAPER 2**

QUESTION ONE (30 MARKS)

a. Solve the linear PDE $p+3q=5z+\tan(y-3x)$ (5 marks)

b. Derive a PDE by eliminating the arbitrary function ϕ from the equation $\phi(x^3+y^3+z^3, z^3-2x^2y^2)$ (6 marks)

c. A semi-infinite bar (extending from $x=0$ to $x=\infty$) with insulated sides is initially at the uniform temperature $u=0^0C$. At time $t=0$, the end at $x=0$ is brought to $u=100^0C$ and held there. Use Laplace transform to find the temperature distribution in the bar as a function of x and t. (10 marks)

d. Find the equation of the surface satisfying the equation $4yzp+q+2y=0$ and passing through $y^2+z^2=1, x+z=2$. (9 marks)

Question TWO (20 marks)

a. Find the general solution of $\begin{aligned}\frac{dx_1}{dt}&=\frac{1}{2}x_1+\frac{1}{2}x_2\\ \frac{dx_2}{dt}&=\frac{-3}{2}x_1+\frac{5}{2}x_2\end{aligned}$ (10 marks)

b. Use the method of separation of variables to solve the initial value problem

$u_x=2_t+u$ subject to $u(x,0)=6e^{-3x}$ (10 marks)

Question THREE (20 marks)

a. Derive a PDE by eliminating the arbitrary constants a and b from $z=ax^2+by^2+ab$. (5 marks)

b. A string of length L is stretched between points $(0,0)$ and $(L,0)$ on the x axis. At time $t=0$ it has a shape given by $f(x), \quad 0\le x\le L$ and it is released from rest. Find the displacement of the string at any latter time. (15 marks)

Question FOUR (20 marks)

a. Show that the Laplace's equation $\nabla^2 u=0$ is satisfied by the function $u=\frac{1}{r}$

where $$u=\frac{1}{\left[(x-x_o)^2+(y-y_0)^2+(z-z_0)^2\right]^{\frac{1}{2}}}$$ (6 marks)

b. Solve the interior Dirichlet problem for a rectangle defined by Laplace's equation

PDE: $\nabla^2 u=0$, $0\le x\le a$, $0\le y\le b$ subject to the boundary conditions

BC's: $u\,(x,0)=u(a,y)=0$, $u\,(0,y)=0$, $u\,(x,b)=0, u\,(x,0)=f(x)$ (14 marks)

Question FIVE (20 marks)

a. Show that the orthogonal trajectories on the hyperboloid $x^2 + y^2 - z^2 = 1$ of a conic in which it is cut by the system of planes $x + y = c$ are the curves of intersection with the family of surfaces $(x - y)z = k$ where k is a parameter. (13marks)

b. Find the integral curves of the equations $\dfrac{dx}{x^2 - y^2 - z^2} = \dfrac{dy}{2xy} = \dfrac{dz}{2xz}$ (7 Marks)

A SHORT TABLE OF LAPLACE TRASFORMS

$f(t)$	$L\{f(t)\}$
1	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+1}$
$\frac{\sin at}{t}$	$\tan^{-1}\frac{a}{s}$ $for\ \text{Re}\, s > ima$
$\sin at$	$\frac{a}{s^2 + a^2}$, $for\ \text{Re}\, s > ima$
$\cos at$	$\frac{s}{s^2 + a^2}$, $for\ \text{Re}\, s > ima$
$\frac{1}{t}\sin at \cos bt$	$\frac{1}{2}\left(\tan^{-1}\frac{a+b}{s}\right) + \tan^{-1}\left(\frac{a-b}{s}\right)$ $for\ \text{Re}\, s > 0$
$1 - erf\left(\frac{a}{2\sqrt{t}}\right), a > 0$	$\frac{1}{s}e^{-a\sqrt{s}}$, $\text{Re}\, s > 0$