TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATION FOR:

AMA 5106: TEST OF HYPOTHESIS

END OF SEMESTER EXAMINATION

SERIES: MAY 2016

TIME: 3 HOURS

DATE: MAY

Instructions to Candidates
You should have the following for this examination
-*Answer Booklet, examination pass and student ID*
This paper consists of five questions. Attempt any three.
Do not write on the question paper.

Question ONE

a. Let $x_1, x_2, \ldots, x_n$ be independently identically distributed $bin(1, p)$ random variable. Find a most powerful size α for $\begin{matrix} H_0; p = p_0 \\ H_1; p = p_1 \end{matrix}$ where p_0 and p_1 are specified $(p_1 > p_0)$ (7marks)

b. Show that the 1 parameter exponential family $f(x;\theta) = \exp\{\Theta(\theta)T(x) + D(\theta) + S(x)\}$ has a Monotone Likelihood Ratio. (5 marks)

c. Let the vector of random variables $x = (x_1, x_2, \ldots, x_n)$ have the probability mass function $f(x;\theta)$ where $\{f(x;\theta), \theta \in \Omega\}$ have a monotone likelihood ratio $T(x)$. Show that for testing $H_0 : \theta \le \theta_0$ against $H_1 : \theta > \theta_0$ any test of the form $\phi(x) = \begin{cases} 1 & if \quad T(x) > t_0 \\ \nu & if \quad T(x) = t_0 \\ 0 & if \quad T(x) < t_0 \end{cases}$ has a non-decreasing power function and is uniform most powerful test. (8marks)

d. Define a consistent test (4 marks)

e. Define a uniformly most powerful test (6marks)

Question TWO

a. Show that if a sufficient statistics T exists for the family $\{f(x;\theta), \theta \in \Omega\}$ $\Omega = \{\theta_0, \theta_1\}$ then the Neyman- Pearson Most powerful test is a function of T. (10 marks)

b. The heat evolved in calories per gram of a cement mixture is approximately normally distributed. The mean is thought to be 100 and the standard deviation is 2. We wish to test $H_0; \mu = 100$ versus $H_1; \mu \neq 100$ with a sample of n = 9 specimens.

   i. If the acceptance region is defined as $98.5 \leq \bar{x} \leq 101.5$, find the type I error probability (3 marks)

   ii. Find the type two error for the case where the true mean heat evolved is 103 . (3marks)

   iii. Find the power of the test for the case where the true mean heat evolved is 105. This value (4 marks)

Question THREE

a. Define the likelihood ratio test (7 marks)

b. Show that if $\{f(x;\theta), \theta \in \Omega\}$ admits a sufficient statistics T then for testing $H_0; \theta \in \Omega_0$ against $H_1; H_0; \theta \in \Omega - \Omega_0$ likelihood ratio test a function of the sufficient statistics. (3marks)

c. Let $x_1, x_2, \ldots, x_n$ be independently identically distributed $N(\mu, \sigma^2)$ random variables. Find a size α likelihood ratio test for testing $H_0; \mu = \mu_0$ against $H_1; \mu \neq \mu_0$ (10 marks)

Question FOUR

a. Let $X \sim bin(n, p)$ if $n \to \infty$ and p is close to Let $\frac{1}{2}$, find a size Let α approximate uniform most powerful unbiased test for $\begin{matrix} H_0; p = p_0 \\ H_1; p = p_1 \end{matrix}$ against (10 marks)

b. Let $x_1, x_2, \ldots, x_n$ be independently identically distributed $N(0, \sigma^2)$ random variables. Determine a uniform most powerful unbiased test for the hypothesis of the form $H_0; \sigma^2 = \sigma_0^2$ against $H_1; \sigma^2 = \sigma_1^2$ (10 marks)

Question FIVE

a. Let $x_{i1}, x_{i2}, \ldots, x_{in}$ be independently identically distributed $N(\mu_i, \sigma_i^{\,2})$ random variables for $i = 1, 2, \ldots, k$. Find a size α LRT test for $H_0; \mu_i = \mu_j$ against $H_1; \mu_i \neq \mu_j$ (15 marks)

b. Show that for testing $H_0; \theta_1 \le \theta \le \theta_2$ against $H_1; \theta < \theta_1$ or $\theta > \theta_2$ there exists a uniform most powerful unbiased size α test given by $\phi(x) = \begin{cases} 1 & if \quad T(x) > c_1 \\ v & if \quad T(x) = c_2 \\ 0 & if \quad c_1 < T(x) < c_2 \end{cases}$ (5 marks)