TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATION FOR:

AMA 5106: TEST OF HYPOTHESIS

END OF SEMESTER EXAMINATION

SERIES: MAY 2016

TIME: 3 HOURS

DATE: MAY

Instructions to Candidates

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

This paper consists of five questions. Attempt QUESTION ONE and any other TWO.

Do not write on the question paper.

Question ONE

a. State and prove Neyman-Pearson Lemma (8 marks)

b. A manufacturer is interested in the output voltage of a power supply used in a PC. Output voltage is assumed to be normally distributed, with standard deviation 0.25 Volts, and the manufacturer wishes to test $H_0; \mu = 5$ Volts against $H_1; \mu \neq 5$ Volts, using 8 units.

i. The acceptance region is $4.85 \leq \bar{x} \leq 5.15$ Find the level of significance. (4marks)

ii. Find the power of the test for detecting a true mean output voltage of 5.1 Volts. (5marks)

c. Show that the class of all test functions is a convex function (3marks)

d. Define the power function of a test (4marks)

e. Show that 1-parameter exponential family has a monotone likelihood ratio. (6marks)

Question TWO

a. Let x be a random variable with probability density function $f(x)$. Find a size α test for; (7marks)

$$H_0; f(x) = f_0(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$$

$$H_1; f(x) = f_1(x) = \frac{1}{\pi}\frac{1}{1+x^2}$$

b. Let $x_1, x_2, \ldots, x_n$ be independently identically distributed $N(0, \sigma^2)$ random variables. Determine whether there exists a uniform most powerful test for the hypothesis of the form $H_0; \sigma^2 = \sigma_0^2$ against $H_1; \sigma^2 = \sigma_1^2$ (8 marks)

c. Show that for testing $H_0; \theta_1 \le \theta \le \theta_2$ against $H_1; \theta < \theta_1$ or $\theta > \theta_2$ there exists a uniform most powerful unbiased size α test given by $\phi(x) = \begin{cases} 1 & if \quad T(x) > c_1 \\ v & if \quad T(x) = c_2 \\ 0 & if \quad c_1 < T(x) < c_2 \end{cases}$ (5 marks)

Question THREE

a. Define an unbiased test (5 marks)

b. If the pdf $f(x;\theta)$ are such that the power function of every test is continuous and if ϕ_0 is uniform most powerful among all tests satisfying some conditions and is level α test, then show that ϕ_0 is unbiased. (5 marks)

c. Let $X \sim bin(n, p)$, find an unbiased size α test for $\begin{matrix} H_0; p = p_0 \\ H_1; p = p_1 \end{matrix}$ against (10 marks)

Question FOUR

a. Let $x_1, x_2, \ldots, x_n$ be independently identically distributed $N(\mu, \sigma^2)$ random variables, Let $y_1, y_2, \ldots, y_n$ be independently identically distributed $N(\mu, \sigma^2)$ random variables. Where σ^2 is common. Suppose $X'_i\, s$ and $Y'_i\, s$ are independent. Determine a size α LRT test for $H_0; \mu = \mu_0$ against $H_1; \mu \ne \mu_0$ (10 marks)

b. Let $x_{i1}, x_{i2}, \ldots, x_{in}$ be independent normally distributed random variables with mean μ_i and variance σ_i^2. Determine a α likelihood ratio test for the hypothesis of the form $H_0; \sigma_i^{\,2} = \sigma_j^2$ against $H_1; \sigma_i^{\,2} \ne \sigma_j^2$ (10 marks)

Question FIVE

a. Determine a α likelihood ratio test for the hypothesis of the form $H_0; \sigma^2 = \sigma_0^2$ against $H_1; \sigma^2 = \sigma_1^2$ (μ is unknown) (10marks)

b. Let $y_1, y_2, \ldots, y_n$ be independently identically distributed $N(\beta, \theta^2)$ random variables. Find a size α likelihood ratio test for testing $H_0; \beta = \beta_0$ against $H_1; \beta \ne \beta_0$ (10 marks)