TECHNICAL UNIVERSITY OF MOMBASA

SCHOOL OF APPLIED AND HEALTH SCIENCES

MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATION FOR:

UNIT: CONTINUUM MECHANICS

UNIT CODE: AMA 4437

END OF SEMESTER EXAMINATION

SERIES: MAY SERIES

TIME: 2HOURS

Instructions to Candidates
You should have the following for this examination
-*Answer Booklet, examination pass and student ID*
This paper consists of five questions. Attempt Question one and any other two.
Do not write on the question paper.

Question ONE

a). Show that $\frac{\partial Ap}{\partial X^q}$ is not a tensor even though Aρ is a tensor of rank one. (5mks)

b). Determine metric tensor in:
i. Cylindrical co-ordinates
ii. Spherical co-ordinates (6mks)

c). Differentiate between Body forces and Surface forces giving an example of each. (4mks)

d). If the velocity component of a 2-D flow is given by

$$U(x/y) = \frac{k(x^2-y^2)}{x^2+y^2} \qquad V(x/y)= \frac{2kxy}{x^2+y^2}$$

Show that the flow is incompressible. (6mks) e). Define:

i. Normal Stress (2mks)

ii. Shear Stress (2mks)

f). In a 3-D incompressible fluid the velocity component in x & y direction and

U=$x^2 + y^2$

V= x +$yx + yz$

Use continuity equation to evaluate an expression for the velocity component in x-direction. (5mks)

Question TWO

a). Prove that:

i. $\frac{\partial x^p}{\partial X^{-q}} \quad \frac{\partial x^{-q}}{\partial X^r} = \delta_r^p$ (3mks)

ii. δ_r^p is a mixed tensor of rank 2 (4mks)

b). Show that the contraction of the other multiplication of the tensor A^p and B_q is an invariant. (6mks)

c). A quantity A (p, q, r) is such that in the co-ordinate system X^q

A (p, q, r) $B_r^{qs} = C_p^s$ when B_r^{qs} is an aborting tensor and C_p^s is a tensor. Prove that A (p, q, r) is a tensor. (7mks)

Question THREE

1. In a 3-D incompressible flow the velocity component in z and w directions are:

V= $ax^3 - by^2 + cz^2$ W = $bx^3 - cyz + az^2x$

a) Determine the missing component of velocity distribution so that the continuity equation is satisfied. (6mks)

b) Verify if the velocity component satisfies the continuity equation.

U= $2x^2 + 3y$ V= $-2xy + 3y^2 + 3zy$ W=$-{}^3/_2 z^2 - 2xz - 6yz$ (5mks)

c) The velocity vector of an incompressible flow is given by

V= $(6xt + yz^2)$i + (3t + xy^2)j + (xy-2xyz-6tz) k

i. Determine the acceleration at a point P(2, 2, 2) (4mks)

ii. Verify if it satisfies the continuity equation (5mks)

Question FOUR

a). Discuss the flow for which w=z^2 (5mks)

b). If Q=A $(x^2 - y^2)$ represent a possible flow phenomena. Determine the stream function. (4mks)

c). Determine the stream function φ (x, y, t) for the given velocity field.

U= ut V= x

U= $-\frac{\partial\varphi}{\partial y}$ V= $\frac{\partial\varphi}{\partial x}$ (7mks)

d). If the potential of stream function is described by:

$$\varphi = x^3 - 3xy^2$$

Determine whether the flow is rotational or irrotational (4mks)

Question FIVE

a). The tensor D is given by the algebraic equation D=A:B. Obtain the order of the tensor D and its components for the following cases.

i. When $A_{ij} = \begin{vmatrix} -2 & 3 & 2 \\ 4 & 1 & 1 \\ 1 & 1 & 5 \end{vmatrix}$, $B_{ij} = \begin{vmatrix} 2 & 3 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 5 \end{vmatrix}$ (4mks)

ii. When $A_{ik}B_{qj} = \begin{vmatrix} 7 & 13 & 14 \\ 11 & 18 & 11 \\ 16 & 27 & 31 \end{vmatrix}$, $A_{ik}B_{jk} = \begin{vmatrix} 13 & 9 & 17 \\ 15 & 9 & 13 \\ 18 & 12 & 32 \end{vmatrix}$ (4mks)

b). Starting from the fundamental equation of continuum mechanics, obtain the growing equation for a rigid solid problem. (12mks)