TECHNICAL UNIVERSITY OF MOMBA

FACULTY OF APPLIED AND HEALTH SCIENCES

MATHS AND PHYSICS DEPARTMENT

UNIVERSITY EXAMINATION FOR:

BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

AMA 4435: MEASURE INTEGRATION AND PROBABILITY PAPER 1

END OF SEMESTER EXAMINATION

SERIES: MAY 2016

TIME: 2 HOURS

DATE: MAY 2016

<u>Instructions to Candidates</u>
You should have the following for this examination
-*Answer Booklet, examination pass and student ID*
This paper consists of FIVE questions. Attempt QUESTION 1 AND ANY OTHER TWO FROM QUESTIONS 2- 5.
Do not write on the question paper.

Question ONE (30 MARKS)

a. Define the following terms

   I. Measure of a set (2 marks)

   II. A space (2 marks)

   III. A complete measure (2 marks)

b. Distinguish between the positive and negative parts of a function (4marks)

c. Let $(\mathcal{X}, x)$ be a measurable space, if $x \subseteq X$ is called a σ- algebra, outline the conditions that it must be satisfy.(4 marks)

d. By use of a counter example, show that if $f \subset x$, so that if and f and $f \in x$ but the converse is not true (6 marks)

e. State the monotone convergence theorem(4 marks)

f. State three examples of measurable spaces (3 marks)

g. Outline three conditions that make a measurable set to be countable.(3marks)

Question TWO (20 marks)

a. Let ($\mathfrak{X}$, $\mathfrak{x}$) be a measurable space. In order that a function f:$\mathfrak{X} \to R_e$ be $\mathfrak{x}$- measurable. Outline the necessary and sufficient conditions that must be fulfilled (8marks)

b. Let ($\mathfrak{X}$, $\mathfrak{x}$) be a measurable and f, g: $\mathfrak{X} \to \mathbb{R}_e$ be $\mathfrak{x}$- measurable functions and let $c \in \mathbb{R}$. Prove that the functions $cf, c+f,\ f^2, |\ f|,\ f+g,\ fg\ f^+\ and\ f^-\ are\ all$ $\mathfrak{x}$- measurable (12 marks)

Question THREE

a. Let f: $\mathfrak{X} \to R_e$ be measurable with$f \gg 0$ (f:$\mathfrak{X} \to (0, \infty)$). Prove that there exists an increasing sequence of simple functions $\emptyset_n : X \to \mathbb{R}$ which converges to f pointwise. i.e. f(x) = $\lim_{n\to\infty}(\emptyset_n(x))^1$ for all $x \in \mathbb{R}$ (12 marks)
b. Prove that if a function f is measurable then a measurable function is integrable *iff* |f | is integrable and $|\int f du| \ll \int |f du|$ (8 marks)

Question FOUR (20 marks)

a. Let ψ be asimple measurable function belonging to $M^+(\mathfrak{X}, \mathfrak{x})$ Let $\lambda : \to R_e$ be defined by $\lambda(E) = \int \psi\ \chi_E d\mu \quad for\ E \epsilon$ $\mathfrak{x}$. Show that $\lambda(E)\ is\ a\ measure$ (14 marks)
b. State Fatou's lemma. Do not prove it. (4 marks)
c. State the law of large numbers.

Question FIVE (20 marks)

a. What do you understand by the term probability measure? State four points. (4 marks)
b. State Demorgan's laws and prove that a set is enclosed under countable intersections (6 marks)
c. Let ($\mathfrak{X}$, $\mathfrak{x}$, μ) be a measure space. prove that μ is monotone if $E, F \epsilon$ $\mathfrak{x}$ $and\ F \subset E$ (4 Marks)
d. State and prove the central limit theorem (6marks)