



# TECHNICAL UNIVERSITY OF MOMBASA

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## SCHOOL OF APPLIED AND HEALTH SCIENCES

### MATHEMATICS AND PHYSICS

### UNIVERSITY EXAMINATION FOR:

### UNIT: ANALYTICAL MECHANICS

UNIT CODE: AMA 4434

### END OF SEMESTER EXAMINATION

### SERIES: MAY SERIES

TIME: 2HOURS

#### Instructions to Candidates

You should have the following for this examination

-Answer Booklet, examination pass and student ID

This paper consists of five questions. Attempt Question one and any other two.

**Do not write on the question paper.**

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#### Question ONE

(a) Show that the ratio of gravitational to electrostatic forces between two electrons is about 1042. Given  $1/4\pi \epsilon_0 = 9 \times 10^{19} \text{NM}^2/\text{C}^2$   $e = 1.6 \times 10^{19} \text{c}$   $G = 6.7 \times 10^{11} \text{NM}^2/\text{kg}^2$   $m_e = 9.1 \times 10^{31} \text{kg}$   
(4mks)

Define the term

- i. Rigid body
- ii. Mechanical constraint

(4mks)

(c) Show that the motion of one projectile as seen from another projectile will always be straight

(6mks)

(d) A body is allowed to slide on a frictionless track from rest position under gravity. The track ends in a circular loop of diameter  $D$ . find the minimum height of the body in terms of  $D$  so that it may complete successfully the loop.

(5mks)

(e) A mass is revolving in a vertical circle at the end of a string of length 4 metres. Calculate the difference in kinetic energy at the top and bottom of the circle. Prove that the tension of the string at the lowest point exceeds the tension at the top most point by  $6Mg$ .

(5mks)

(f) A rocket starts from rest with an initial mass  $M_0$  and reaches a final velocity  $V_f$  at burnout when its mass is  $M_f$ .

i. Show that

$$M_f/M_0 = \exp[-V_f/V_{ex}]$$

Where  $V_{ex}$  is the exhaust velocity

ii. Show that the rocket speed is equal to the exhaust speed when the ratio  $M_0/M$  is  $e$  and equal to twice the exhaust speed when  $M_0/M$  is  $e^2$

(6mks)

### Question TWO

(a) Define

i. A closed system

ii. Internal and external forces

(6mks)

(b) A two stage rocket is employed to carry a payload of 50kg to an altitude  $R/4$  above earth's surface. The radius of the earth is  $R=6000\text{km}$ . the first stage of rocket is 5 times heavier as compared to the second stage and total mass ratios for both are 10. Find the minimum total mass required of exhaust velocity is 1500m/s. Neglect atmosphere drag and rotation of earth that you may assume that both the stages of rockets are fired in quick succession.

(14mks)

### Question THREE

(a) What do you understand by the term rigid body

(2mks)

(b) Give two examples of rigid bodies (2mks)

(c) A uniform disc of radius  $R$  and mass  $M$  is mounted on an axle supported in affixed frictionless bearing. A body of mass  $M$  hangs from the cord wrapped around the rim of the disc. Find the angular acceleration and the tangential acceleration of a point on a rim (7mks)

(d) A metal stick is held vertically with one hand on the floor and then is allowed to fall. Find the speed of the other hand when it hits the floor. Assuming that the hand of the floor doesn't slip. (7mks)

#### Question FOUR

(a) In a two body oscillator the masses of the two bodies are equal can we think it as if the connecting wire were cut two halves and each mass were oscillating independently. (4mks)

(b) A horizontal platform with an object placed on it is executing SHM in vertical direction. The amplitude of oscillating is 2.5cm what must be the least period of these oscillations so that the object is not detached from the platform. Take  $g=10\text{m/s}^2$  (6mks)

(c) A uniform spring whose unstressed length is  $l_1$  has a force constant  $k$  the spring is cut into two pieces of unstressed length  $l_1$  and  $l_2$  where  $l_1=n l_2$ ,  $n$  being an integer. What are the corresponding force constants  $k_1$  and  $k_2$  in terms of  $n$  and  $k$  (10mks)

#### Question FIVE

(a) What will be the Hamiltonian of a particle moving with a potential  $U$  in a uniformly rotating frame of reference (4mks)

(b) What is a Hamiltonian function of a holonomic system having  $n$  degrees of freedom (5mks)

(c) Construct the Lagrangian and the equation of motion of a spherical pendulum placed in a uniform gravitation field. (11mks)