**TECHNICAL UNIVERSITY OF MOMBASA**

*A Centre of Excellence*

*Faculty of Applied & Health Sciences*

**DEPARTMENT OF MATHEMATICS AND PHYSICS**

**UNIVERSITY EXAMINATION FOR THE SECOND SEMESTER IN THE FOURTH YEAR OF BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE**

**MAY 2016 SERIES EXAMINATION**

**UNIT CODE: AMA 4426**

**UNIT TITLE: STOCHASTIC PROCESSES**

**TIME ALLOWED: 2HOURS**

***INSTRUCTIONTO CANDIDATES:***

You should have the following for this examination

- Mathematical tables
- Scientific Calculator

This paper consists of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

**Maximum marks for each part of a question are as shown**

---

## QUESTION ONE (30 MARKS)

(a) Define the following:

(i) A stochastic process (2 marks)

(ii) A Bernoulli process (2 marks)

(b) Let Y have a geometric distribution given by

$$P(Y=k)=\begin{cases} q^k p; & k=0,1,2,3,\ldots.. \\ 0\ ; & elsewhere \end{cases}$$

Find (i) the probability generating function of Y (4 marks)

(ii) the mean and variance of Y (6 marks)

(c) . Let $\{X_n : n \geq 0\}$ be a Markov chain with three states 0,1,2 and transition probability matrix

$$\begin{pmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{pmatrix}$$

And the initial probability distribution $P(X_0=i)=\begin{cases} \frac{1}{4}, i=0 \\ \frac{1}{3}, i=1 \\ \frac{5}{12}, i=2 \end{cases}$

Find :

(i) $P(X_2=2\,,\ X_1=1\,/X_0=2)$ (3marks)

(ii) $P\ (X_1=1\,/X_0=2)$ (1 mark)

(iii) $P\ (X_2=2\ /\ X_1=1)$ (1 mark)

(iv) $P\ (X_3=1\,,\ X_2=2\,, X_1=1\,, X_0=2)$ (3 marks) (d). The joint distribution of two random variables X and Y is given by:

$$P_{jk} = P\{X = j, Y = k\} = \begin{cases} q^{j+k}p^2, & j = 0,1,2,\ldots, k = 0,1,2,\ldots, \quad p + q = 1 \\ 0 & otherwise \end{cases}$$

Obtain the:

(i). bivariate p.g.f of X and Y (4 marks)

(ii). P.g.f of X (2 marks)

(iii). P.g.f of X+Y (2 marks)

## QUESTION TWO (20 MARKS)

(a) Define the following terms :

(i) Irreducible Markov chain (2 marks)

(ii) Persistent state (2 marks)

(iii) A periodic state (1 mark)

(iv) Ergodic state (1 mark)

(b). A markov chain with state space $\{E_1, E_2, E_3\}$ has the following probability transition matrix

$$\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$$

Classify the states of the process. (14 marks)

## QUESTION THREE (20 MARKS)

Consider a population whose size at time t is Z(t) and let the probability that the population size is n be denoted by $P_n(t) = P\{Z(t) = n\}$ with $P_1(0) = 1$ *and* $P_n(0) = 0$ , $n \neq 1$ . Further let :

(i) The chance that an individual produces a new member in time t interval $\Delta t$ be $\lambda \Delta t$ where $\lambda$ is some constant be n.

(ii) The chance of an individual producing more than one member be $0(\Delta t)$ (i.e negligible).

(a) Show that $P_n(t) = e^{-\lambda t}\{1 - e^{-\lambda t}\}^{n-1}$ , $n \neq 1$

(b) Find the second raw moment of the process

(20 marks)

## QUESTION FOUR (20 MARKS)

(a) Explain the following terms:

(i) A strictly stationary stochastic process (2 marks)

(ii) A covariance stationary process (2 marks)

(iii) An evolutionary process (2 marks)

b) (i) Write down the differential-difference equations for the Polya process. Hence obtain the probability generating function given that $P_n(0) = 1$ *when* $n = 0$ *and* $P_n(0) = 0$ *when* $n \neq 0$

(ii) Show that the Polya process is not covariance stationary.

(14 marks)

## QUESTION FIVE (20 MARKS)

Consider the difference-differential equations for the Poisson process given by

$$P'_n(t) = \begin{cases} -\lambda P_n(t) + \lambda P_{n-1}(t) & : \ n \geq 1 \\ -\lambda P_0(t) & : \ n = 0 \end{cases}$$

With initial conditions $P_0(0) = 1 \ when \ n = 0 \ and \ P_n(0) = 0 \ when \ n \neq 0$

(i) Find the solution of the equation.

(ii) Use Feller's method to find the mean and variance of the process.

(20 marks)