



# TECHNICAL UNIVERSITY OF MOMBASA

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Faculty of applied and Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

## UNIVERSITY EXAMINATION FOR:

BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

AMA 4323: ORDINARY DIFFERENTIAL EQUATIONS II

END OF SEMESTER EXAMINATION

**SERIES:** MAY 2016

**TIME:** 2 HOURS

**DATE:** 2016

**PAPER B**

### Instructions to Candidates

You should have the following for this examination

-Answer Booklet, examination pass and student ID

This paper consists of 5 questions. Question one is compulsory. Answer any other two questions

**Do not write on the question paper.**

### **QUESTION ONE (COMPULSORY)**

(a) Solve the system of linear equations

$$x'(t) = 3x(t) - 4y(t)$$

$$y'(t) = 4x(t) - 7y(t)$$

(6 marks)

(b) (i) Consider the equation  $y' = f(x, y)$ . Define a rectangular region  $L$  by

$|x - x_0| \leq a$  and  $|y - y_0| \leq b$  With points  $(x_0, y_0)$  at its centre. Then there exists a function

$\phi(x)$  On the interval  $|x - x_0| \leq h$ . Give the four properties of  $\phi(x)$  (4 marks)

(ii) State the existence and uniqueness theorem (2 marks)

(iii) Show the convergence of the initial value problem

$$\frac{dy}{dx} = y; \quad x_0 = 0, \quad y_0 = 1 \quad (5 \text{ marks})$$

(c) Reduce the third order equation below to a first order system of equations

$$\frac{d^3 y}{dx^3} + 8 \frac{d^2 y}{dx^2} - 10 \frac{dy}{dx} + 7y = 2x^3 \quad (4 \text{ marks})$$

(d) Define the following terms

(i) Total differential equation (2 marks)

(ii) Ordinary point (2 marks)

(e) Find the values of  $x$  and  $y$  in the first order system

$$\frac{dx}{dt} = y, \quad \frac{dy}{dt} = -2x + 3y \quad (5 \text{ marks})$$

## QUESTION TWO

(a) Solve  $(yz + xyz)dx + (zx + xyz)dy + (xy + xyz)dz = 0$  (13 marks)

(b) Locate and classify the singular points of the equation

$$(x^2 - 8x) \frac{d^2 y}{dx^2} + (x + 2) \frac{dy}{dx} + y = 0 \quad (7 \text{ marks})$$

## QUESTION THREE

(a) Solve  $X' = AX$  where  $X = \begin{pmatrix} x \\ y \end{pmatrix}$  and  $A = \begin{pmatrix} 2 & -5 \\ 2 & -4 \end{pmatrix}$  (10 marks)

(b) Find two independent series solutions of the Legendre equation

$$(1 - x^2)y'' - 2xy' + 2y = 0 \quad (10 \text{ marks})$$

#### QUESTION FOUR

- (a) Obtain the indicial equation of

$$16x^2 y'' + (x+2)y = 0 \quad (8 \text{ marks})$$

- (b) Solve  $\frac{d^3 y}{dx^3} = xe^x$  (9 marks)

- (c) What are the three conditions for the exactness of

$$Pdx + Qdy + Rdz = 0 \quad (3 \text{ marks})$$

#### QUESTION FIVE

- a) Consider the first order vector equation  $X'(t) = Ax(t) + B(t)$  where  $A$  is an  $n \times n$  matrix of real numbers,  $x(t)$  is a column vector function of  $t$  and  $B(t)$  an  $n$ -dimensional column vector function of  $t$ . show that its characteristic polynomial is given by  $|A - mI|$  (13 marks)

- b) Determine the singular points of the equation

$$(x^2 - 81) \frac{d^2 y}{dx^2} + 9x \frac{dy}{dx} + (x+9)y = 0 \quad (5 \text{ marks})$$

- c) What is a differential equation (2 marks)