TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHELOR OF TECHNOLOGY AND RENEWABLEL ENERGY & BACHELOR OF TECHNOLOGY AND APPLIED PHYSICS

AMA 4109: CALCULUS FOR TECHNOLOGISTS 1 :

END OF SEMESTER EXAMINATION

SERIES: APRIL 2016

TIME: 2 HOURS

DATE: Pick Date May 2016

QUESTION ONE (COMPULSORY 30MKS)

a) Find the derivative of the following

    i). $y = sin^2 x$ (3marks).

    ii) $y = \tanh^{-1} x$ (4 marks)

iii). $y = Cot(\tan x)$ (4 marks)

b). Show that $\tanh^2 x + \sec h^2 x = 1$ (4 marks

c). A rectangular sheet of metal of length 6 meters and width 2 meters is given. Four equal

squares of sides x meters are removed from the corners. The sides of this sheet are now turned up to form an open rectangular box. Find appropriately the height of the box which will give the maximum volume of the box. (7marks)

d).Evaluate the following limits

    i) $\lim_{x \to 2} \frac{x^2 - 5x + 6}{x - 2}$ (4 Marks)

    ii) $\displaystyle Lim_{x \to 49} \frac{x - 49}{6 - \sqrt{x - 13}}$ (4 marks)

QUESTION TWO (20MKS)

a).Given a parametric function $x = 1 - t^2$, $y = t^2 - t$ .Determine $\frac{dy}{dx}$ at t=2. (4marks).

b). Find the turning points on the curve of $y = x^3 - 6x^2 + 9x - 2$ and distinguish between them.
(10 marks)

c).Determine the asymptotes to the curve $y = \frac{(x-1)(x+6)}{(x+3)(x-4)}$. (6marks)

QUESTION THREE (20MKS)

a).If $y^2 + x^2 = 2y\sqrt{(1 + x^2)}$ $show\ that$ $\frac{dy}{dx} = \frac{x}{\sqrt{(1 + x^2)}}$, (5marks)

b).Determine the area enclosed by the functions $y = x^3 + 2x^2 - 5x - 6$, $x = -3$, $x = 2$ and the x-axis (5 marks)

c).By use of the product rule, determine $\frac{d^2y}{dx^2}$ of y= x^2sin2x (10marks)

QUESTION FOUR (20MKS)

a).Determine the equation of the normal to the curve $x^2y + y^2x - x^3 + y^2 = 9$ at (1,2)
(6marks)

b).Evaluate

        $(i).\lim_{x \to 0} \frac{1 - \sqrt{(1 - x)}}{x}$ (4marks)

        $(ii)\lim_{x \to 0} \frac{5 + x}{x^2 + x + 1}$ ⋮   (3marks)

c). Differentiate $y = \cos x$ from first principals (7marks

QUESTION FIVE (20MKS)

a).Define continuity.   (3marks)

b).Gas is escaping from a spherical balloon at a rate of $0.01\text{m}^3/\text{s}$. How first is the radius changing when the surface area is 10 m². (6marks).

c). Prove that $\dfrac{d}{dx}\left(\dfrac{1+\sin x+\cos x}{1-\sin x+\cos x}\right)=\dfrac{1}{1-\sin x}$ (5marks

d).Determine

$$\lim_{x\to\infty}\frac{\tan x-x}{x^3}$$ (6 marks)