# TECHNICAL UNIVERSITY OF MOMBASA <br> UNIVERSITY EXAMINATIONS 

BFSQ/BTAC
B
AMA4109: CALCULUS FOR SCIENCES
INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO

## QUESTION ONE (30MARKS)

(a) Integrate the function $f(x)=3 x^{2}-2 x+5$.
(b) (i) Define what is meant by $f: X \longrightarrow Y$ is a continuous function at $x=a$.
(ii) Investigate continuity of $f(x)$ at $x=-1$ and $x=1$ where

$$
f(x)=\left\{\begin{array}{l}
2-x, x<-1 \\
x,-1 \leq x<1 \\
4, x=1 \\
4-x, x>1
\end{array} .\right.
$$

(c) Find $\frac{d^{3} y}{d x^{3}}$ if $y=2 x e^{x}$
(d) Evaluate the following limits
(i) $\lim _{x \rightarrow 0} \frac{\sqrt{x^{2}+16}-4}{x^{2}}$.
(ii) $\lim _{x \rightarrow \infty} \frac{1-\sqrt{x}}{1+\sqrt{x}}$.
(e) Find the area bounded by $x=4-y^{2}$ and the $y$-axis
(f) Decompose the rational fraction $\frac{5 x+2}{(x+2)(3 x-2)}$
(g) Use first principles to differentiate $f(x)=x^{2}+2 x$. Hence find $f^{\prime}(2)$.

## QUESTION TWO (20MKS)

(a) Find the derivatives of
(i) $g(x)=\left(x^{3}-3 x^{2}+6 x+12\right)^{5}$
(ii) $f(x)=\frac{3 x-3}{4 x+9}$
(iii) $h(x)=\ln \left(x^{6}+4\right)$
(b) Use differentials to approximate the value of $\sqrt[6]{1.97}$
(c) Differentiate $f(x)=\sqrt{x-1}$ and hence find
(i) the domain of $f^{\prime}(x)$.
(ii) the tangent line at $x=10$.

## QUESTION THREE (20MKS)

(a) Differentiate the function $y=\sqrt{x} \sinh ^{-1} x^{2}$
(b) Find the vertical asymptotes of $f(x)=\frac{x}{x^{2}-x-2}$.
(c) Show that $\frac{d}{d x}(\operatorname{cosec}-1 x)=-\frac{1}{x \sqrt{x^{2}-1}}$
(d) Find the equation of tangent to the curve $x=3 t^{2}+1, y=2 t^{3}+1$ that passes through $(4,3)$
(e) Find the linearization of $f(x)=\sqrt{x+3}$ at $x=1$ and use it to approximate $\sqrt{3.8}$.

## QUESTION FOUR (20MKS)

(a) Integrate by substitution
(i) $\int \sqrt[3]{1-3 x} d x$
(ii) $\int \frac{(\ln x)^{2}}{x} d x$
(iii) $\int \frac{1}{x^{2}+9} d x$
(b) Integrate by parts
(i) $\int x^{4} \ln x d x$
(ii) $\int x^{3} e^{x} d x$

## QUESTION FIVE (20MKS)

(a) Find all intervals where $f(x)=\frac{1}{3} x^{3}-3 x^{2}+5 x-4$ is increasing or decreasing.
(4mks)
(b) Differentiate
(i) $f(x)=e^{x^{2}} \cosh 4 x$.
(ii) $y=\frac{\sin x}{1+\cos x}$.
(c) A ladder 10 m long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of $1 \mathrm{~m} / \mathrm{s}$, how fast is the top of the ladder sliding the wall when the bottom of the ladder is 6 m from the wall?
(d) Find the arc length of on $y=\frac{x^{3}}{6}+\frac{1}{2 x}$ on $\frac{1}{2} \leq x \leq 2$.

