# TECHNICAL UNIVERSITY OF MOMBASA UNIVERSITY EXAMINATIONS BFSQ/BTAC AMA4109: CALCULUS FOR SCIENCES INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO

# **QUESTION ONE (30MARKS)**

(a) Integrate the function 
$$f(x) = 3x^2 - 2x + 5$$
. (2mks)

(b) (i) Define what is meant by  $f: X \longrightarrow Y$  is a continuous function at x = a. (2mks) (ii) Investigate continuity of f(x) at x = -1 and x = 1 where

$$f(x) = \begin{cases} 2 - x, x < -1 \\ x, -1 \le x < 1 \\ 4, x = 1 \\ 4 - x, x > 1 \end{cases}$$
(4mks)

(c) Find 
$$\frac{d^3y}{dx^3}$$
 if  $y = 2xe^x$  (3mks)

(d) Evaluate the following limits

(i) 
$$\lim_{x \to 0} \frac{\sqrt{x^2 + 16 - 4}}{x^2}$$
. (3mks)

(ii) 
$$\lim_{x \to \infty} \frac{1 - \sqrt{x}}{1 + \sqrt{x}}.$$
 (3mks)

(e) Find the area bounded by  $x = 4 - y^2$  and the y-axis (4mks)

(f) Decompose the rational fraction 
$$\frac{5x+2}{(x+2)(3x-2)}$$
 (4mks)

(g) Use first principles to differentiate  $f(x) = x^2 + 2x$ . Hence find f'(2). (5mks)

# **QUESTION TWO (20MKS)**

(i) 
$$g(x) = (x^3 - 3x^2 + 6x + 12)^5$$
 (3mks)

(ii) 
$$f(x) = \frac{3x-3}{4x+9}$$
 (3mks)

(iii) 
$$h(x) = \ln(x^6 + 4)$$
 (2mks)

(b) Use differentials to approximate the value of  $\sqrt[6]{1.97}$  (5mks)

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#### **QUESTION THREE (20MKS)**

(a) Differentiate the function  $y = \sqrt{x} \sinh^{-1} x^2$  (3mks)

(b) Find the vertical asymptotes of 
$$f(x) = \frac{x}{x^2 - x - 2}$$
. (4mks)

(c) Show that 
$$\frac{d}{dx}(\cos ec^{-1}x) = -\frac{1}{x\sqrt{x^2 - 1}}$$
 (4mks)

(d) Find the equation of tangent to the curve  $x = 3t^2 + 1$ ,  $y = 2t^3 + 1$  that passes through (4, 3)

(5mks)

(e) Find the linearization of 
$$f(x) = \sqrt{x+3}$$
 at  $x = 1$  and use it to approximate  $\sqrt{3.8}$ . (6mks)

# **QUESTION FOUR (20MKS)**

(a) Integrate by substitution (i)  $\int \sqrt[3]{1-3r} dr$ 

(i) 
$$\int \sqrt[3]{1-3x} dx$$
 (3mks)

(ii) 
$$\int \frac{(\ln x)^2}{x} dx$$
 (3mks)

(iii) 
$$\int \frac{1}{x^2 + 9} dx$$
 (5mks)

(b) Integrate by parts

(i) 
$$\int x^4 \ln x \, dx$$
 (4mks)

(ii) 
$$\int x^3 e^x dx$$
 (5mks)

# **QUESTION FIVE (20MKS)**

# (a) Find all intervals where $f(x) = \frac{1}{3}x^3 - 3x^2 + 5x - 4$ is increasing or decreasing. (4mks)

(b) Differentiate

(i) 
$$f(x) = e^{x^2} \cosh 4x$$
. (2mks)  
 $\lim_{x \to \infty} \frac{\sin x}{\cos x}$ 

(ii) 
$$y = \frac{\sin x}{1 + \cos x}.$$
 (4mks)

(c) A ladder 10m long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1m/s, how fast is the top of the ladder sliding the wall when the bottom of the ladder is 6m from the wall?

(d) Find the arc length of on 
$$y = \frac{x^3}{6} + \frac{1}{2x}$$
 on  $\frac{1}{2} \le x \le 2$ . (6mks)