TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHELOR OF MATHEMATICS AND COMPUTER SCIENCE & BACHELOR OF SCIENCE IN STATISTICS AND COMPUTER SCIENCES

AMA 4209: **CALCULUS III**

END OF SEMESTER EXAMINATION

SERIES:APRIL2016

TIME:2HOURS

DATE: Pick DateMay2016

Instructions to Candidates

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

This paper consists of **FIVE** questions. Attempt question ONE (Compulsory) and any other TWO questions.

Do not write on the question paper.

QUESTION ONE (COMPULSORY, 30Marks)

a) Find the $\lim\limits_{t\to\infty}\dfrac{t^2+t}{2t^2+1}$ (2 marks)

b) Two stationary patrol cars with radars are 5km apart on a high way and a truck passes the first patrol car, its speed is clocked at 55km/h. Four minutes later, when the truck passes the second patrol car, its speed is clocked at 50km/h. Prove that the truck must have exceeded the speed limit of 60 km/h at some point during the interval. (3 marks)

c) Apply the integral test to the series $\sum_{n=1}^{\infty}\dfrac{n}{n^2+1}$. To determine divergence or convergence

(5marks)

d) Use Maclaurin theorem to expand the function $f(x)=e^{2x}$ upto the term with x^5.

(5 marks)

e).Determine convergence/divergence of the series $\sum_{n=0}^{\infty} \frac{n^2 2^{n+1}}{3n}$ using ratio test.(5mks).

f) The equation $xz + y\ln x - x^2 + 4 = 0$ defines x as a differentiable function of two independent variables y and z, find $\frac{\partial x}{\partial y}$, $\frac{\partial x}{\partial z}$ at the point (1,-1,-3). (6 marks)

g) Find the rectangular form of the polar function $r = 2\cos 2\theta$ (4 marks)

QUESTION TWO (20 Marks)

a)Evaluate $\lim_{x\to\infty} \frac{\sqrt{3x^2+6}}{5-2x}$ (5marks).

b).Test whether the sequence $\{a_n\}$ where $\{a_n\}=\frac{n^2}{(n+1)^2}$ is convergence and find its limit (5 Marks)

c).Find the value of $\frac{df}{dt}$ at $t = 0$ if $f(x,y,z) = xy + z$ and $x = \cos t$, $y = \sin t$, $z = t$. (5 marks)

d).Find the area of the region R bounded by $y = x$ and $y = x^2$ in the first quadrant. (5 marks)

QUESTION THREE (20MKS)

a) Calculate the volume bounded by $f(x, y) = 1 - 6x^2 y$ on the region $R: 0 \le x \le 2, \quad -1 \le y \le 1.$ (4 marks)

b) Find the Taylor polynomials P_0, P_1, P_2, P_3 and P_4 for $f(x) = \ln x$ contained at $c = 1$. (6 marks)

c) Evaluate $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$. (6 marks)

d) Find the sum of the geometric series $\sum_{n=0}^{\infty} \frac{3}{2^n}$. (4 marks)

QUESTION FOUR (20 Marks)

a) Find the rectangular coordinates corresponding to the polar coordinate $\left(2, \frac{2\pi}{3}\right)$. (4 marks)

b).The probability density function $f(x) = \frac{t}{1+x^2}$ has the area under the curve in the interval

$(-\infty, \infty)$ Equals to 1. Determine the values of t . (8marks)

c).). Determine if the function is convergent or divergent

$$\int_0^3 \frac{1}{\sqrt{3-x}}\,dx$$ (4 marks)

d) Find the total differential of $z = x^3y + x^2y^2 + xy^3$ (4 marks)

QUESTION FIVE (20 Marks)

a) Find the radius of convergence of the power series $\sum_{n=0}^{\infty} 3(x-2)^n$ (4 marks)

b) Find a sequence $\{a_n\}$ whose first five terms are $\frac{2}{1}, \frac{4}{3}, \frac{8}{5}, \frac{16}{7}, \frac{32}{9}, \ldots$ and determine whether it converges or diverges. (6 marks)

c) Let R be the square $\{(x, y) \mid -1 \le x \le 1, \quad -1 \le y \le 1\}$. Calculate the volume of the solid region determined by the graph of $f(x, y) = 8 - x^2 - y^2$ over R. (6 marks)

d) Evaluate $\lim\limits_{x \to 0} \frac{\tan x - x}{x^3}$ (4 marks)