TECHNICAL UNIVERSITY OF MOMBASA

END OF SEMESTER EXAMINATION

AMA 4212 VECTOR ANALYSIS

QUESTION ONE (3OMKS)

a) Given $\vec{r}_1 = 3\hat{i} - 2\hat{j} + \hat{k}, \vec{r}_2 = 2\hat{i} - 4\hat{j} - 3\hat{k}, \hat{r}_3 = -\hat{i} + 2\hat{j} + 2\hat{k}$, find $|\vec{r}_1 + \vec{r}_2 + \vec{r}_3|$ (4mks)

b) Find the area of a triangle having vertices $p(1,3,2), Q(2,-1,1), R(-1,2,3)$ (4mks)

c) Find an equation of the plane perpendicular to the vector $\vec{A} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ and passing through the terminal point of the vector $\vec{B} = \vec{i} + 5\hat{j} + 3\hat{k}$ (4mks)

d) Find a unit normal to the surface $x^2 y + 2xz = 4$ at the point $(2,-2,3)$ (4mks)

e) Given $F = (2xy + z^3)i + x^2 j + 3xz^2 k$

i. Show that F is a conservative force field (2mks)

ii. Find the work done in moving an object in this field from $(1,-2,1)$ to $(3,1,4)$ (4mks)

f) Verify green's theorem in the plane for $\oint_c (xy + y^2)dx + x^2 dy$ where C is a closed curve of the region bounded by $y = x, y = x^2$ (4mks)

g) Evaluate $\iint_R xydxdy$ over the region R is the region bounded by x-axis, ordinate $x = 2a$ and the curve $x^2 = 4ay$ (4mks)

QUESTION TWO (2OMKS)

a) If $\vec{A} = 5t^2\hat{i} + t\hat{j} + -t^3\hat{k}$ and $B = \sin t\hat{i} - \cos t\hat{j}$ Find :

i. $\frac{d}{dt}(A.B)$ (4mks)

ii. $\frac{d}{dt}(A \times B)$ (4mks)

b) Find the directional derivative of $\phi = x^2 yz + 4xz^2$ at $(1,-2,-1)$ in the direction $2i - j - 2k$

(4mks)

c) Evaluate $\iiint_v Fdv$ where $\vec{F} = xy\hat{i} + z\hat{j} - x^2\hat{k}$ and V is the region bounded by surfaces $x = 0, x = 2, y = 0, y = 3, z = 0, z = 4$ (4mks)

d) Find the constants a,b,c so that $v=(x+zy+az)\hat{i}+(bx-3y-z)\hat{j}+(4x+cy+2z)\hat{k}$ is irrotational. (4mks)

QUESTION THREE (2OMKS

a) Find an equation for tangent plane to the surface $2xz^2-3xy-4x=7$ at the point $(1,-1,2)$ (4mks)

b) Find the angle between the surfaces $x^2+y^2+z^2=9$ and x^2+y^2-3 at the point $2,-1,3$ (6mks)

c) The acceleration of a particle at any time $t\geq 0$ is given by $a=12\cos 2t\hat{i}-8\sin 2t\hat{j}+16t\hat{k}$. If the velocity v and displacement r are zero at $t=0$. Find v and r at any time (6mks)

d) Find a unit tangent vector to any point on the curve $x=a\cos\omega t, y=a\sin\omega t, z=bt$ where a,b,ω are constants. (4mks)

QUESTION FOUR (2OMKS

a) Find the constant a such that the vectors $2\hat{i}-\hat{j}+\hat{k}, i+2\hat{j}-3\hat{k}, 3\hat{i}+a\hat{j}+5\hat{k}$ are coplanar. (4mks)

b)

i. State the Gauss divergence theorem (3mks)

ii. Verify the divergence theorem for $A=(2x-z)i+(x^2y)j-xz^2k$ taken over the region bounded by $x=0, x=1, y=0, y=1, z=0, z=1$ (13mks)

QUESTION FIVE (2OMKS)

a) Determine the constant a so that the vector $V=(x+3y)\vec{i}+(y-2x)\vec{j}+(x+az)\vec{k}$ is soleinodal. (5mks)

b) Given the space curve $x=t, y=t^2, z=\frac{2}{3}t^2$ find,

i. The unit tangent T (4mks)
ii. The principal normal N (4mks)
iii. The curvature k (2mks)
iv. The radius of curvature (2mks)
v. The binormal B (3mks)