TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED SCIENCES
MATHEMATICS AND PHYSICS DEPARTMENT

UNIVERSITY EXAMINATION FOR BACHELOR OF TECHNOLOGY DEGREE IN APPLIED PHYSICS (BTAP) AND BACHELOR OF TECHNOLOGY DEGREE IN RENEWABLE ENERGY (BTRE)

APS 4301: WAVE THEORY AND TIDAL ENERGY

END OF SEMESTER EXAMINATION

SERIES: May Series 2016:

TIME: 2 HOURS **DATE: May 2016**

Instructions to Candidates

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

This paper consists of **FIVE** questions. Attempt Question **ONE** and any other **TWO** questions.

Do not write on the question paper.

The maximum marks for each question is shown.

Mathematical tables and scientific calculators may be used.

The following constraints may be useful: Gravitation acceleration, g = 9.89 m/s^2

QUESTION ONE (30 MRKS)

a) (i). Name two types of waves. (2mks)

(ii). Give any four properties that are used to characterize a wave. (2mks)

(iii). Mention one significance of Fourier theorem as used in waves. (1mk)

b) (i) When is a motion said to be in simple harmonic? (1mk)

(ii) State the principle of superposition of waves. (2mk)

c) A mass, M, was suspended from a table on a spring with spring constant 16 N/m as shown in figure 1 below. It was displaced slightly to execute simple harmonic motion with varying amplitude given as $\boldsymbol{x = A\cos(\omega t + \theta)}$ with a maximum amplitude of A = 0.05m**;**

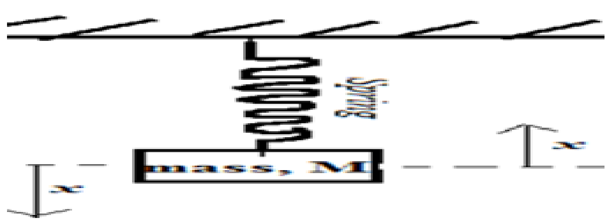

Figure 1: Hanging spring

(i) Derive an expression for its maximum potential energy. (5mks)
(ii) Calculate its maximum potential energy. (2mks)
(iii) Calculate the maximum kinetic energy of the system. (3mks)
(iv) Compare the maximum kinetic energy and maximum potential energy. (2mks)

d) A student at TUM connected two particles of similar mass of M by a spring of constant k_{12} and further connected each particle to fixed points with other two springs of constant, k_1 and k_2. She restricted the particles to move only along the x-axis with two degrees of freedom x_1 and x_2 respectively from equilibrium position as shown in the figure 2.

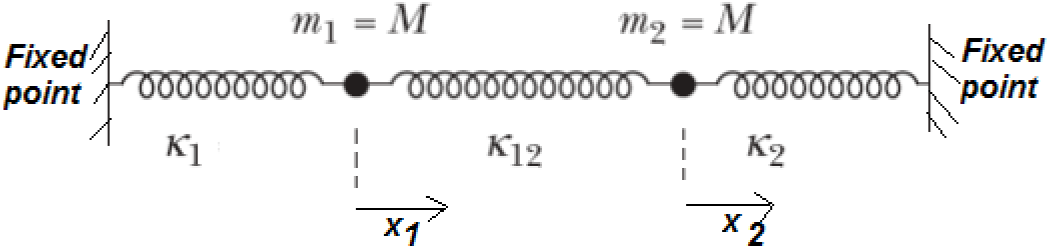

Figure 2: Two masses connected by springs

Show that if the springs have $k_1 = k_2 = k$, characteristic frequencies of the system.

$$\omega_1 = \sqrt{\frac{k + 2k_{12}}{M}} \text{ and } \omega_2 = \sqrt{\frac{k}{M}}$$ (10 mks)

QUESTION TWO (20 MKS)

a) Show that a generalized wave equation for a travelling wave given by $\boldsymbol{y = A \sin(\omega t + \theta)}$ in the **x**-axis with linear velocity, $\boldsymbol{v}$, angular frequency $\boldsymbol{\omega}$, and phase angle, $\boldsymbol{\theta}$ can be expressed as;

$$\frac{1}{v^2} = \frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 x}{\partial x^2}$$ (6mrks)

b) (i) State the principle of superposition of waves. (1mrks)
(ii) A system two coupled linear oscillators are connected to display normal modes as shown.

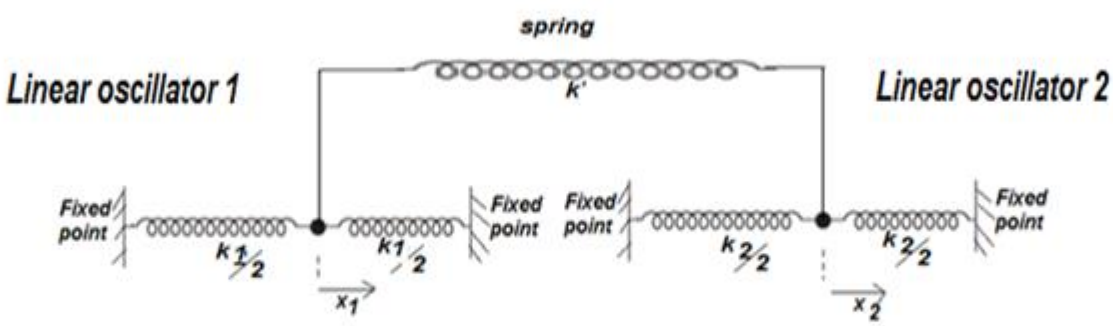

Figure 3: Two linear oscillators connected by a spring k'

(i) If both linear oscillators are released from rest with the same displacement, show that their resonant angular frequency is $\omega_1 = \sqrt{\frac{k}{m}}$ and spring constant ***k'*** is not present in the solution expression for ω_1. (6mks)

(ii) If one oscillator is released from rest while the other is released from rest but with a displacement, show that, $\omega_2 = \sqrt{\frac{k + 2k'}{m}}$ is the angular frequency of the oscillators system. (5mks)

(ii) Calculate their coupled period, T if ***k*** is 4N/m and ***k'*** is 3.6N/m with mass of oscillator m is 0.02kg. (2mrks)

QUESTION THREE (20 MRKS)

a) (i) Define the term Wave power. (1mrk)

(ii) State the Fourier Theorem for waves. (3mrks)

(iii) Give one implication of Fourier transformation on waves. (2mrks)

(iv) Differentiate between Kinetic energy and potential energy of a wave. (2mrk)

b) The spring shown in figure 4 below was initially a free vibrating spring executing simple harmonic motion. If a sinusoidal force, $F = F_o \sin \omega t$ was added.

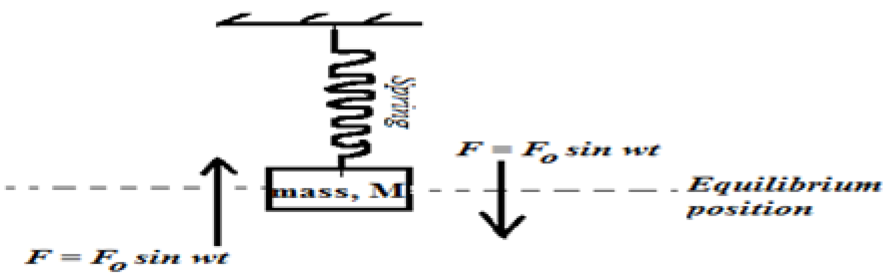

Figure 4: Spring carrying a mass, M

(i) Show that the superposed oscillation by the added force can be expressed as

$\frac{\partial^2 x}{\partial t^2} + \omega_o^2 x - \frac{F_o}{m}\cos\omega = 0$ where ω_o is the frequency of the spring. (3mrks)

(ii) If this force introduced a damping term '**bv**" onto the spring, show that the new amplitude can expressed as $A = \frac{F_o}{m\sqrt{(\omega_o^2 - \omega^2) - (\gamma\omega^2)}}$ where $\gamma = \frac{b}{m}$ and $w^2 = \frac{k}{m}$ of added force. (6mrks)

c) A simple pendulum with a bob of mass, M and length *l* was displaced along the x-axis with a small angle θ as measured from the equilibrium and allowed to execute uniform simple harmonic motion. Derive an expression to show itsperiodic time, T . (4mrks)

QUESTION FOUR (20 MKS)

a) A ferry of mass, M and of floor cross-sectional area, **A** floating in a Likoni's deep ocean water of density ρ. If ***l*** is the length of ferry below the surface of water and a small force by a container is applied by a dropping the container onto the ferry causing the ferry to depress into the ocean water, a distance *x* resulting into uniform simple harmonic motion, show that the period, T, of the ferry will be given by $T = 2\pi\sqrt{\frac{l}{g}}$ (5mrks)

(ii) What will be the effect on the period of this ferry if the length of the ferry is increased; when the cross-sectional area of the ferry is increased and when the density of the liquid is increased. (3mrk)

(iii) Given that the rod in (a) above has a length of 50m, a cross-sectional area of 600 square meters and the density of the sea water is $1.KgM^{-3}$, determine its period of oscillation. (2mrks)

b). Consider a flexible elastic string to which are attached ***n*** identical particles each of mass, M, placed a distance *L* apart. If the initial tension in the string is T, derive an expression to show the resultant force on any typical particle when oscillating as a many coupled harmonic oscillator. Hence show the expression for the angular frequency of the **n^{th}** mode of the particle. (10mrks)

QUESTION FIVE (20 MKS)

a) (i) Define the term resonance. (2mrks)

(ii) Where can we observe resonance? (1mrks)

(iii) Using a sketch, differentiate between light damping and heavy damping. (4mrks)

b) A tidal wave moves solid particles up and down as it propagates. Consider the motion of such a solid particle being carried by a tidal wave. Let the tidal wave be assumed to be acting as if it is in a certain circular path of radius A and centre **O** as shown in figure 5 below. If its displacement is given by $\boldsymbol{x = A\cos(\omega t + \Phi)}$. Derive an expression for its maximum velocity along the ***x***-axis. (4mrks)

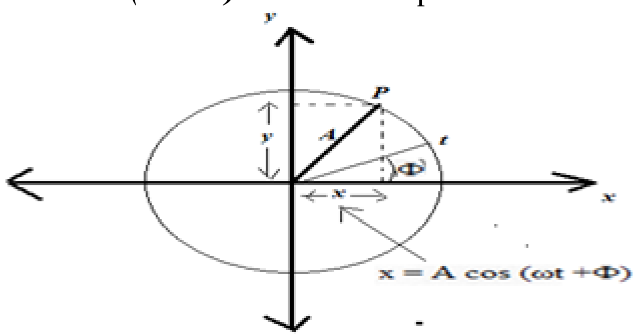

Figure 5

c). (i) How are tidal waves generated in deep seas waters? (3mrks)

(ii) Describe how an overtopping device is used in generating tidal power. (2mrks)

d) Calculate the power of a wave having a wave height of 3m and a period of 8 seconds. (4mrks)

END