



TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHALOR OF MATHEMATICS AND COMPUTER SCIENCE

SMA4318: COMPLEX ANALYSIS 1

END OF SEMESTER EXAMINATION

SERIES: APRIL 2016

TIME: 2 HOURS

DATE: May 2016

Instructions to Candidates

You should have the following for this examination

-Answer Booklet, examination pass and student ID

This paper consists of **FIVE** questions. Attempt question ONE (Compulsory) and any other TWO questions.

Do not write on the question paper.

Question ONE

- (a) Evaluate $\lim_{z \rightarrow \infty} \frac{2z^2 + 4z + 5}{5z + z - 5}$ (3mrks)
- (b) If $a = 6 - 3i$ and $b = 2 - i$, Evaluate $\frac{a+b}{a}$ (3mrks)
- (c) Find the conjugate of the complex number $Z = \frac{1}{2 - 3i}$ (3mrks)
- (d) Evaluate $\lim_{z \rightarrow 3 + 4i} \left(\frac{4 + z^2}{z} \right)$ (3mrks)
- (e) If $f(z) = (z^3 + 4z)^2$ find the derivative of $f(z)$ (4mrks)
- (f) show that $u(x, y) = 2x - x^3 + 3xy^2$ is harmonic (4mrks)
- (g) prove that $\sec(z) = \frac{z}{e^{iz} + e^{-iz}}$ (4mrks)
- (h) Find all the singular points of the function $f(z) = \frac{i + z^3}{2 - 3z + z^2}$ (4mrks)
- (i) Describe the domain of the function $f(z) = \frac{z}{e^z - 1}$ (2mrks)

Question TWO

- (a) Show that the function $f(z) = \frac{-1}{(z-1)(z-2)}$ is analytic in the domain $D_1, : |z| < 2$ and $D_2, : 1 \leq |z| < 2$ (6mrks)
- (b) Evaluate $\int \frac{1}{4 + z^2} dz$ where $c: |z - i| = 2$ (7mrks)
- (c) Find the Taylor series for the function $f(z) = e^{2z}$ (7mrks)

Question THREE

- (a) Solve for z in $e^z = -1$ (5mrks)
- (b) Prove that $\sin(iy) = i \sinh(y)$ (5mrks)
- (c) Show that $\log(1+i)^2 = 2 \log(1+i)$ (5mrks)
- (d) Evaluate $(-i)^i$ (5mrks)

Question FOUR

- (a) If $z = 2 + 5i$ find $z^{\frac{1}{4}}$ the fourth roots of z (7mrks)
- (b) Illustrate the following transformation $f(z) = z^2$ for the line $x=1$ (7mrks)
- (c) Show that $f(z) = \frac{z}{z}$ is not continuous on the entire Z -plane (3mrks)
- (d) Evaluate $2i^i$ (3mrks)

Question FIVE

- (a) Find the derivative of $f(z) = z^2$ by Cauchy theorem (6mrks)
- (b) Show that the function $f(z) = 3x + y + (3y - x)i$ is entire (4mrks)
- (c) Find the Harmonic conjugate of the function $u(x,y) = e^y \sin(x)$
Hence find the function $f(x,y) = u + iv$ (6mrks)
- (d) Show that the function in $f(z) = \frac{2z+4}{5z}$ is continuous at $z=2$ (4mrks)