

TECHNICAL UNIVERSITY OF MOMBASA

Faculty of applied and Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONIC ENGINEERING

SMA 2480: COMPLEX ANALYSIS

END OF SEMESTER EXAMINATION

SERIES: MAY 2016

TIME: 2 HOURS

DATE: 2016

PAPER A

Instructions to Candidates

You should have the following for this examination -Answer Booklet, examination pass and student ID This paper consists of 5 questions. Question one is compulsory. Answer any other two questions **Do not write on the question paper.**

QUESTION ONE (COMPULSORY)

a) Given that
$$z_1 = 2 + i$$
, $z_2 = 3 - 2i$, $z_3 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$ evaluate
i) $|3z_1 - 4z_2|$ (3 marks)

ii) $\left|\frac{2z_2 + z_1 - 5 - i}{2z_1 - z_2 + 3 - i}\right|^2$ (4 marks)

b) Derive the Polar form of complex numbers from a point say A(x, y) on the Cartesian plane.(6 marks)

c) Prove that $(\cos \theta + i \sin \theta)^n = (\cos n\theta + i \sin n\theta)$ where *n* is a positive integer (3 marks)

d) Obtain the isolated singular points , $\operatorname{Res}{f(z), a}$ of the function given by

$$f(z) = \frac{1}{(z-3)(z+1)}$$
 (6 marks)

- e) Define a Laplace inverse transform (3 marks)
- f) Show that if images to two curves under a conformal mapping are orthogonal then the curves are orthogonal
 (2 marks)
- g) Check if the function $z^2 = (x^2 y^2) + 2xyi$ satisfies the Cauchy Riemann equations (3 marks)

QUESTION TWO

a) State and prove the Cauchy Riemann Equations (10 marks)

b) Evaluate
$$\iint_{c} \frac{e^{z}}{(z+1)^{2}} dz$$
 where *c* is the circle $|z-1|=3$ (6 marks)

c) Show that $e^{iz} = \cos z + i \sin z$ (4 marks)

QUESTION THREE

- a) State and prove the Residue Theorem (10 marks)
- b) Using definition of a derivative, obtain the derivative of $w = f(z) = z^3 2z$ at

$$z = z_0 = 0$$
 (10 marks)

QUESTION FOUR

a) State and prove the Cauchy Integral Theorem (12 marks) b) Evaluate $\int_{0}^{4+2i} z dz$ along the curve given by $z = t^2 + it$ (8 marks)

QUESTION FIVE

- a) Given that $L\{f(x)\} = \frac{s+1}{s^2+s-6}$ obtain f(x) (12 marks)
- b) Show that $u = e^{-x}(x \sin y y \cos y)$ is a harmonic function (8 marks)