TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

AMA 4410: PARTIAL DIFFERENTIAL EQUATIONS 1

END OF SEMESTER EXAMINATION

SERIES:APRIL2016

TIME:2HOURS

DATE: Pick DateMay2016

Instructions to Candidates

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

This paper consists of **FIVE** questions. Attempt question ONE (Compulsory) and any other TWO questions.

Do not write on the question paper. **PAPER 1**

QUESTION ONE (30 MARKS)

a) Describe the orthogonal trajectories of $y = kx^2, k \neq 0$ [6 Marks]

b) Obtain the general solution to the partial differential equation
$(y-z)p + (z-x)q = x - y$ [4 Marks]

c) Show that a the partial differential equation arising from

$$z = \frac{1}{2}(a^2+2)x^2 + axy + bx + \phi(y+ax)$$

can be put in the form $(r+u)(t+v) = s^w$ where u, v, w are integers. [6 Marks]

d) Find the direction cosines of the space curve defined by the parametric equations
$x = -0.5s^2, \quad y = 0.25s^3, \quad z = 1.5s^2$ through $(-2,-2,6)$ [6 Marks]

e) Find the complete solution of $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = \sin(3x - y) + 12xy$. [8 Marks]

QUESTION TWO (20 MARKS)

a) Classify the partial differential equation

$$\frac{\partial^2 z}{\partial x^2}+\left(5+2y^2\right)\frac{\partial^2 z}{\partial x \partial y}+\left(1+y^2\right)\left(4+y^2\right)\frac{\partial^2 z}{\partial y^2}=0$$

and find its characteristics. [10 Marks]

b) Find a complete integral of the equation $p^2x+q^2y-z=0$ using Charpit's method. [10 Marks]

QUESTION THREE (20 MARKS)

a) Derive the wave equation $\frac{\partial^2 u}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ for a perfectly flexible vibrating string of uniform density ρ stretched to a uniform density τ between two points $x=0$ and $x=L$; $c^2=\frac{\tau}{\rho}$ [8 Marks]

b) Solve the wave equation in (a) above satisfying the Cauchy conditions

$$u(0,t)=u(L,t)=0, \quad t\geq 0$$
$$u(x,0)=f(x), \qquad 0\leq x\leq 0$$
$$u_t\big|_{t=0}=g(x), \qquad 0\leq x\leq 0$$

where f and g are given functions [12 Marks]

QUESTION FOUR (20 MARKS)

a) Find the General Solution of $\frac{\partial^2 z}{\partial x^2}-2\frac{\partial^2 z}{\partial x \partial y}+5\frac{\partial^2 z}{\partial y^2}=\sin(3x-y)$ [5 Marks]

b) Find a partial differential equation arising from the general solution

$$\phi\left(x^6-y^6, \frac{x^3+y^3}{z^3}\right)=0$$

[5 Marks]

c) Find a complete solution of $p^2x+q^2y=z$ using Jacobi method. [10 Marks]

QUESTION FIVE (20 MARKS)

a) Find the orthogonal trajectories on the conicoid $z(x+y)=4$ of a cone in which it is cut by the system of planes $x-y+z=k$ where k is a parameter. [10 Marks]

b) Find the general integral of the partial differential equation $(2xy-1)p+(z-2x^2)q=2(x-yz)$ and also the particular integral which passes through the line $x=1, \quad y=0$ [10 Marks]