TECHNICAL UNIVERSITY OF MOMBASA

A Centre of Excellence

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

APRIL 2016 SERIES EXAMINATION

UNIT CODE: SMA 2273 UNIT TITLE: APPLIED MATHEMATICS 1

SPECIAL/SUPLIMENTARY EXAMINATION

TIME ALLOWED: 2HOURS

INSTRUCTIONTO CANDIDATES:

You should have the following for this examination

- Mathematical tables
- Scientific Calculator

This paper consists of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

QUESTION ONE (30 MARKS) COMPULSORY

a. For a body in vertical motion, show that maximum height of a projectile is $h_m = \frac{U_0^2 sin^2\Theta}{g}$ (3 marks)

b. State Newton's second law of motion and use it to derive the formula $f = ma$ (3 marks)

c. State four uses of dimensional analysis (4 marks)

d. 2 Forces P and Q which are inclined at 120^0 have a resultant magnitude of P. calculate the magnitude of Q in terms of $P\sqrt{7}$ (4 marks)

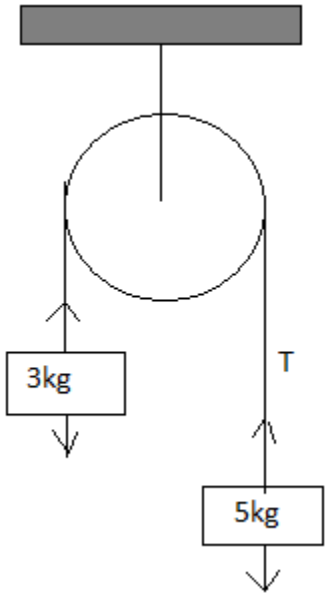
e. A stone of mass 0.4kg is tied to a string of length 0.5 and whirled in a circle. If the stone revolve uniformly and makes one complete revolution per second, calculate its acceleration and the force exerted on the stone by the string (2 marks)

f. Two particles have position vectors given by

$$r_1 = 4ti - 2t^2j - 5tk$$
$$r_2 = (2t^2 - t)i + t^3j - 4tk$$

Find the relative velocity and acceleration of second particle with respect to the first particle when t=3 seconds (6 marks)

g. Two particles of mass 5kg and three kg are connected by a light inelastic string passing over a smooth fixed pulley. find the acceleration of particles and tension in the string when the system is moving freely (3 marks)



h. A coil spring lies along the helix. $r = (cos4t)i + (sin4t)j + tk, 0 \leq t \leq 2\pi$. The spring's density is a constant δ=1. Find spring's mass and spring's moment of inertia and radius of gyration about the z axis (5 marks)

QUESTION TWO(20 MARKS)

a. A particle is moving along a curve defined by the parametric equation $x = 2cos3t$ $y = 2sin3t \quad z = 4t^2$. find
   i. Velocity and acceleration at any time t. (3 marks)
   ii. Show that the speed of the particle is increasing but the magnitude of acceleration is constant. (4 marks)

b. A particle p is projected from point o on a horizontal plane with a speed of 72km/h at an angle θ to the horizontal where $tan\,\Theta = \frac{4}{3}$. Find
   i. Time taken for P to return to the plane (2 marks)
   ii. Maximum height attained by P (2 marks)
   iii. The range (2 marks)
   iv. Speed of P after two seconds. (2 marks)

c. A particle on a circle of radius R has a constant angular acceleration α. If the particle starts from rest, show that after a time t
   i. The angular velocity $\omega = \alpha t$ (2 marks)
   ii. The magnitude of acceleration $a_T \; and a_N$ (3 marks)

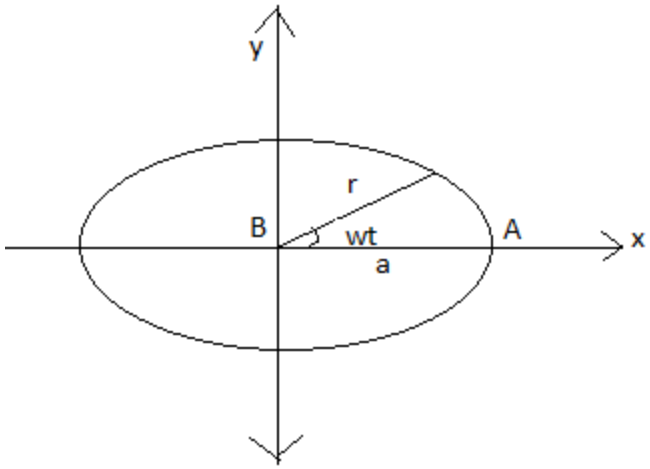
QUESTION THREE (20 MARKS)

a. A block of mass 2kg is kept moving with a uniform acceleration of 0.2m/s^2 by an application of force of 10.4N. What was the limiting frictional force? (3 marks)

b. A particle is fired with a constant velocity of $10x10^5 m/s$ into the region where it is subjected to an acceleration of $2x10^{12}$m/s^2 directed to the initial velocity. How far does the particle travel before coming to rest? How long does the particle remain at rest? (3 marks)

c. A particle moves with position function$r(t) = (t^2, t^2, t^3)$. Find
   i. The unit tangent vector at t=1 (3 marks)
   ii. The tangential and normal acceleration to the point. (3 marks)

d. A particle of mass m kg moves in the x y axis plane so that its position vector is where a, b and w are positive constants and a>b
   i. Show that the force field is conservative (3 marks)
   ii. Find the potential energy at the points A and B in the figure below (2 marks)



iii. Find the work done by the force in moving the particle from A to B (1 marks)

iv. Find the total energy of the particle and show that it is constant, i.e. demonstrate the principle of conservation of energy. (2 marks)

QUESTION FOUR (20 MARKS)

a. A particle is projected from a point which is 2m above the ground level with a velocity of 40m/s at an angle 45 to the horizontal. Find its horizontal distance from the point of projection when it hits the ground. (5 marks)

b. A particle of mass 5 units moves along a space curve given by $r = (t^2 + t)i + (3t - 2)j + (2t^3 - 4t^2)k$. find

   i. Velocity o a particle (1 marks)

   ii. Acceleration of the particle (1 marks)

   iii. Force acting on a particle (1 marks)

   iv. Momentum of particle at t=2 (1 marks)

c. A coin is thrown vertically upwards from the ground with a speed of 10m/s.

   i. How long does it take to reach the maximum point (1 marks)

   ii. What is the maximum height reached by the coin? (2 marks)

d. Calculate the resultant of vectors $v_1 - v_2 + v_3$ given that
   $V_1 = 22\ units\ at\ 140^0$
   $V_2 = 40\ units\ at\ 190^0$
   $V_3 = 15\ units\ at\ 290^0$ (4 marks)

e. If $F = (x, y, z) = y^2 j + (2xy + e^{3z})j + 3ye^{3z}k$ find a function f such that $\boldsymbol{\nabla f = F}$

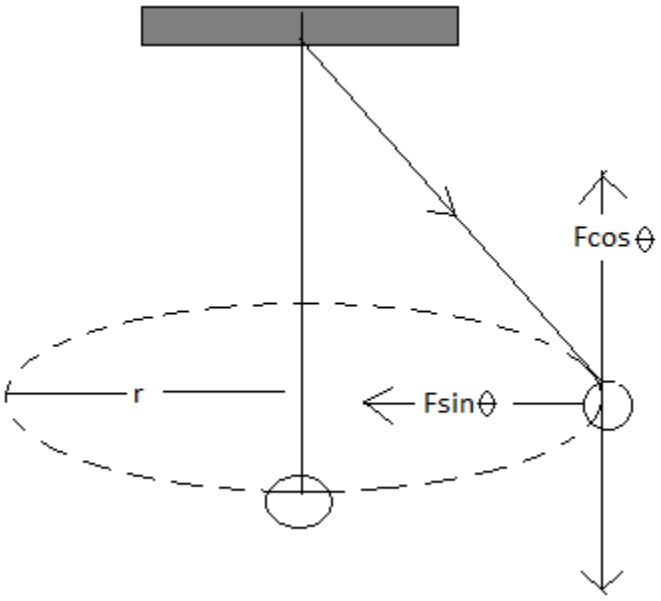
(4 marks)

QUESTION FIVE (20 MARKS)

a. The efficiency η of a fun depends on the density ρ the dynamic viscosity μ of the fluid, the angular velocity ω, diameter D of the rotor and the discharge Q. express in terms of dimensionless parameters (7 marks)

b. Find the work done in moving a particle once around a circle c in the x-y plane with centre origin and radius 3 units by a force given by $F = (2x - y + z)i + (x + y - z^2)j + (3x - 2y + 4z)k$ for $x = 3cos\theta \;\; y = 3sin\,\theta . \theta$ changing from zero to $360(2\pi)$ (6 marks)

c. For a conical pendulum.



Show that for unit radius of the circular path $tan\Theta = \frac{v^2}{g}$ (3 marks)

d. A particle of unit mass moving a straight line is acted upon by a force given by -$4xN$, where x is the displacement of 1kg particle. The particle is at rest when x=3 meters. Find the velocity when x=1 (4 marks)

THIS IS THE LAST PRINTED PAGE