TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BTIT

SMA 2230: PROBABILITY & STATISTICS II

END OF SEMESTER EXAMINATION

SERIES:APRIL2016

TIME:2HOURS

DATE:20May2016

Instructions to Candidates

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

This paper consists of Choose No questions. AttemptChoose instruction.

Do not write on the question paper.

Question ONE (30 MarkS)

(a) Define the following terms:

(i) Random variable

(ii) parameter

(iii) Random experiment

(iv) Sample space (4 marks)

(b) Let X be a discrete random variable with distribution

X	0	1	2
P(X=x)	3/8	¼	3/8

Find:

(i) P(X=0 or X=1) (2 marks)

(ii) Mean and variance of X (4marks)

(c) A lot of size 100 contains 50 defective articles. Suppose that a sample of 10 articles is drawn at random from the lot, find:

(i) The probability mass function of the number of defectives, X (2 marks)

(ii) The probability that the sample contains less than 2 defectives(4 marks)

(d) The mean weight of 500 packets of sugar is found to be 1012g. of the 500 packets, 35 were found to have a weight in excess of 1015 g. Assuming the weights are normally distributed about the mean, estimate :

(i) The standard deviation of the weights (3 marks)

(ii) The number of packets weighing less than 1008g (2 marks)

(e) A machine is designed to produce automotive break disks of diameter 120mm and $\sigma = 4mm.$

(i) If a random sample of 40 disks had a mean diameter of 120.97, test at 5% level significance whether the machine is working normally (3 marks)

(ii) Would the conclusion change if a random sample of 10 disks were used instead? (3 marks)

(f) Find the mean, variance and standard deviation of a binomial random variable with n=10, p=0.8 (3 marks)

Question TWO (20 MarkS)

(a) Find the moment generating function of a random variable whose probability density function is given by

$$f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & elsewhere \end{cases}$$ (6 marks)

(b) Using the moment generating function of the random variable in (a), find:

(i) the mean μ (2 marks)

(ii) the second moment μ_2' (2 marks)

(iii) the variance μ_2 (2 marks)

(c) The number of cars, X, that pass through a car wash between 4.00p.m. and 5.00 p.m. on any sunny Friday has the following probability distribution:

X	4	5	6	7	8	9
P(X=x)	1/12	1/12	¼	¼	1/6	1/6

Let g(X)= 2X-1 represent the amount in K£ paid to the attendant by the manager. Find:

(i) The attendant's expected earnings for this particular time (3 marks)

(ii) The variance of the attendant's earnings for the given period (5 marks)

Question THREE (20 MarkS)

(a) Compilation of a computer program consists of 3 blocks that are processed sequentially, one after the other. Each block takes an exponential time with mean of 5 minutes, independently of other blocks. Compute:

(i) The expectation and variance of the total compilation time (4 marks)

(ii) The probability for the entire program to be compiled in less than 12 minutes (7 marks)

(b) The KRA is a body mandated to collect tax on behalf of the Kenya government. If the annual proportion of erroneous tax returns filed with KRA is found to be a random variable having a beta distribution with $\alpha = 6$ and $\beta = 9$, determine:

(i) The mean of erroneous tax returns (4 marks)

(ii) The probability that there will be less than 10% erroneous tax returns (5 marks)

Question FOUR(20 Marks)

(a) A shipment of 7 computers contains 2 computers suspected to be defective.
An IT workshop makes a random purchase of 3 computers. If X is the number of defective computers bought by the workshop;

(i)	Find the probability distribution of X	(2 marks)
(ii)	Express the results graphically on probability histogram	(5 marks)
(iii)	Find the CDF of X	(2marks)

(b) Using (a)(iii), determine:

(i)	P(X=1)	(1 mark)
(ii)	P(0<x<2)	(1 mark)

(c) The time to failure of a certain brand of electric bulb can be represented by the density function

$$f(x) = \begin{cases} \frac{1}{2000} e^{-\frac{1}{2000}x}, & x > 0 \\ 0, & elsewhere \end{cases}$$

Determine:

(i)	F(x)	(2 marks)
(ii)	The probability that the bulb lasts more than 100hours	(1 mark)
(iii)	The probability that the bulb fails before 2000 hours	(1 mark)

Question FIVE(20 Marks)

(a) Define the following terms with regard to hypothesis testing:

(i)	Statistical hypothesis	(1 mark)
(ii)	Type I error	(1 mark)
(iii)	Power of a test	(1 mark)
(iv)	Type II error	(1mark)
(v)	Significance level	(1 mark)

(b) A manufacturer of computer memory modules claims that only 8% of the modules will be defective. An IT equipment distributer buys 20 of these modules from the manufacture. The distributer intends to test:

$H_0: p = 0.08$ against
$H_1: p > 0.08$

Where p is the true proportion of memory modules that are defective. Use x>= 3 as the rejection region, X is the number of defectives.

(i)	Determine the value α for this procedure	(5 marks)
(ii)	Find β, if , infact p= 0.2	(4 marks)
(iii)	Find the power of the test for this value of p	(1 mark)

(c) An important specification in the design a computer processor is the wake-up time. This is the time taken to reach full operation mode from a lo power sleep mode. On a certain type of processor, it is known that the standard deviation of the wake-up time is 2 ns. A quality control supervisor selects a sample of 30 processors from the brand and finds that the average wake-up time is 51 ns. Determine the 95% confidence interval for the mean wake-up time for this brand of processor. (5 marks)