# Faculty of Engineering and Technology Department of Mechanical & Automotive Engineering UNIVERSITY EXAMINATION FOR:

**BSc.** Mechanical Engineering

EMG 2405 : Control Engineering I END OF SEMESTER EXAMINATION

SERIES: APRIL 2016 TIME: 2 HOURS

DATE: Pick Date Apr 2016

#### **Instruction to Candidates:**

You should have the following for this examination

- Answer booklet
- Non-Programmable scientific calculator

This paper consists of **FIVE** questions. Attempt question **ONE** and any other **TWO** questions.

Maximum marks for each part of a question are as shown.

Do not write on the question paper.

### **Question ONE (Compulsory)**

**a.** Figure Q1a shows a process which is being controlled in a closed-loop system with unity feedback. The process can be modelled using a first-order model with system gain, K, and a system time constant,  $\tau$ . (15 marks)

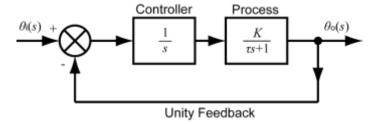


Figure Q1a

- i. Obtain the expression for the forward-path transfer function for the system,  $G_o(s)$ .
- ii. Determine the system type number.
- iii. Obtain the expression for the closed-loop transfer function for the system, G(s).
- iv. If this system was subject to a unit step reference input, what would be the steady-state error value? Use your knowledge of system types, controller effects or the final value theorem.

- v. If the system time constant,  $\tau$  is 0.1 seconds and the system gain K is 2.5, will the closed-loop system be underdamped, critically damped or over-damped?
- b. Consider the closed loop system with a unity feedback as shown in Figure Q1b.

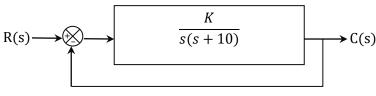


Figure Q1b

- i. Determine the gain K so that the system will have a damping ratio of 0.5.
- ii. For the obtained value of K determine the following for a unit step input:
  - i. Settling time,
  - ii. Rise time,
  - iii. Time to peak,
  - iv. Maximum overshoot (15 marks)

#### **Question TWO**

- a. State any TWO advantages of modern control approach to system modeling as compared to classical approach. (2 marks)
- b. Define the following terms:
  - i. State vector,
  - ii. State variable, (2 marks)
- c. Consider a transfer function of a system as shown in Figure Q2a

$$R(s) \longrightarrow \boxed{\frac{s^2 + 7s + 2}{s^3 + 9s^2 + 26s + 24}} \longrightarrow C(s)$$

- i. Find the state equation and output equation for the phase variable representation of the transfer function.
- ii. Draw an equivalent block diagram showing phase variables. (10 marks)
- d. Figure Q2b shows the translational mechanical system. Obtain the state model of the system. (6 marks)

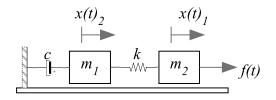


Figure Q2b

## **Question THREE**

a.

i. State Routh-Hurwitz criteria for stability.

(3 marks)

- ii. State any THREE limitations of using Hurwitz criteria for determining stability of a system. (6 marks)
- iii. Consider the closed loop system shown in Figure Q3a. Determine the range of values of K for which the system is stable. (6 marks)

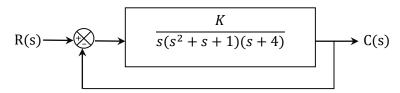


Figure Q3a

b. Figure Q3(b) shows the block diagram for a system which has two subsystem blocks on the feed-forward path and two subsystem blocks on the negative feedback path. Derive an expression for the output response  $\theta_0(s)$  in terms of the reference input  $\theta_i(s)$  and the two disturbance inputs  $\theta_{d1}(s)$  and  $\theta_{d2}(s)$ . (5 marks)

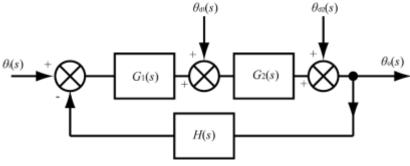


Figure Q3b

## **Question FOUR**

Figure Q4 shows the block diagram of a complex system which includes three negative feedback loops, the outermost of which is a unity feedback loop.

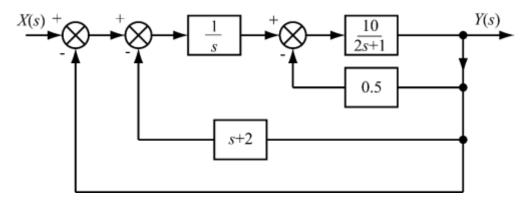


Figure Q4

a. Show whether the above closed-loop system is stable or unstable.

(8 marks)

b. Either through knowledge of the response of system types to different inputs or by applying the final value theorem calculate the steady-state error for this system to the following inputs:

- i. A step input of magnitude 10.
- ii. A ramp input given by x(t) = 10t.
- iii. A parabolic input given by  $x(t) = 10t^2$

(6 marks)

c. Calculate an expression for the output time response of the system y(t) when the input is a step input of magnitude 10 if the output response is zero at time t = 0. (6 marks)

## **Question FIVE**

- a. State the effect of introducing feedback on the stability of control systems. **(2 marks)** b.
  - i. State the Nyquist criterion.
  - ii. The Nyquist plot of a unity feedback system having open loop transfer function is as shown in Figure Q5a.

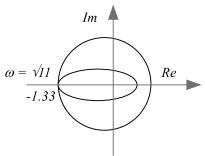


Figure Q5a.

Given the open loop transfer function:

$$G(s) = \frac{K(s+5)(s+3)}{(s-2)(s-4)}$$

When K=1 the system is stable. Determine the range of K for the system to be considered stable. (8marks)

c. Figure Q5b shows a schematic of an armature-controlled d.c. motor which essentially consists of an armature coil in a magnetic field. The armature consists of a resistance, R, and an inductance, L, in series. When current, i, flows through the armature, the coil rotates generating a torque, T, which is proportional to the current, so that  $T = K_m i$ . Since the armature is rotating in a magnetic field, a voltage, known as the back e.m.f. (e) will be induced in it. The back e.m.f. is proportional to the armature rotation speed,  $\omega$ , so that  $e = K_b \omega$ . The motor drives a mechanical load with moment of inertia, I, and with a rotary viscous damping coefficient, c.

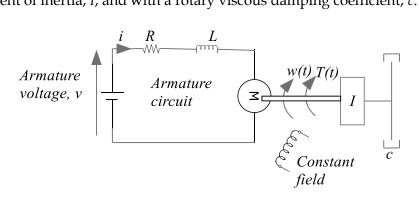


Figure Q5b

Show that the transfer function which relates  $\Omega(s)$ , the Laplace transform of the armature rotation speed,  $\omega(t)$ , to V(s), the Laplace transform of the armature voltage, v(t), is given by the following expression: (10 marks)

$$G(s) = \frac{\Omega(s)}{V(s)} = \frac{K_m}{(LIs^2 + (RI + cL)s + (Rc + K_bK_m))}$$