TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF ENGINEERING & TECHNOLOGY

MECHANICAL ENGINEERING

UNIVERSITY EXAMINATION FOR:

BACHELOR OF SCIENCE

EMG 2414: Numerical Methods for Engineers

END OF SEMESTER EXAMINATION

SERIES: **APRIL 2016**

TIME: **2 HOURS**

DATE: 2016

Instructions to Candidates
You should have the following for this examination
-*Answer Booklet, examination pass and student ID*
This paper consists of Choose No questions. AttemptChoose instruction.
Do not write on the question paper.

Question ONE

(a) Solve each of the following systems of linear equations using Gauss-elimination and state the type of solutions in each.

$4x_1 - 6x_2 = 10$

$6x_1 - 9x_2 = 15$ (2 marks)

$2x_1 + x_2 = 3$

$2x_1 + x_2 = 1$ (3 marks)

(b) Use trapezoidal rule to integrate $\int_0^{\frac{\pi}{3}} \sqrt{\sin x}\,dx$, using six intervals evaluated correct to 3 decimal places

(5 marks)

(c) Consider the initial value problem $y' = x(y+1),\ \ y(0) = 1$. Compute $y(0.2)$ with $h = 0.1$ Using Euler's method. (5 marks)

(d) Using Newton's backward difference formula, find the polynomial for the following data.

x	0	1	2	3
$f(x)$	-3	2	9	18

(5 marks)

(e) Find the Eigen values and Eigen vectors of $A=\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ (5 marks)

(f) Solve the differential equation

$\frac{dy}{dx} = 3e^x - 2y,\ \ y(0) = 0$ by the method of Runge Kutta method of order 4, to get the value of y at $x = 0.1$ given that $h = 0.1$

(5 marks)

Question TWO

(a) Derive the trapezium rule using the Lagrange linear interpolating polynomial for points (a, f(a)),(b, f(b)) (5 marks)

(b) Using the 4th order Runge Kutta method, solve the initial value problem

$\frac{dy}{dx} = -2y + x + 4,\ \ y(0) = 1$ to obtain $y(0.2) = 1$ using $\Delta x = 0.2$ (5 marks)

(c) Using Gauss - elimination solve the system of linear equations. (5 marks)

$x_1 + 3x_2 + 5x_3 = 14$

$2x_1 - x_2 - 3x_3 = 0$

$4x_1 + 5x_2 - x_3 = 7$

(d) Evaluate $\Delta^2 f(x)$, given that $f(x) = 3x^2$, h=0.1 (5 marks)

Question THREE

(a) Use Gaussian Elimination to convert the following matrix into a row echelon matrix

$$\begin{bmatrix} 1 & -3 & 1 & -1 \\ -1 & 3 & 0 & 3 \\ 2 & -6 & 3 & 0 \\ -1 & 3 & 1 & 5 \end{bmatrix}$$

(6 marks)

(b) Using the forward difference calculate $\Delta^2 f(x)$, given that $f(x) = x^2 + 8x - 5$

(7 marks)

(d) Using Taylor series expand $f(x) = \frac{1}{x-1} - 1$ to obtain cubic approximation around $a = 0$

(7 marks)

Question FOUR

(a) Find the Eigen vectors of the matrix $A=\begin{bmatrix}1 & -3 & 3\\ 3 & -5 & 3\\ 6 & -6 & 4\end{bmatrix}$ given that the Eigen values of A are $\lambda=-2, \lambda=-2, \lambda=4,$ (5 marks)

(b) Using Newton's forward difference, find $\frac{dy}{dx}$ at $x=1$ from the following table of value

(5 marks)

x	1	2	3	4
y	1	8	27	64

(c) The velocity of a particle which starts from rest is given by the following table

t	0	2	4	6	8	10	12	14	16	18	20
v(t)	0	16	29	40	46	51	8	32	18	3	0

Evaluate using trapezium rule, the total distance travelled is 20 seconds. (5 marks)

(d) Find $\frac{dy}{dx}$, at x=1.2

x	1	1.2	1.4	1.6	1.8	2.0	2.2
y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

(5 marks)

Question FIVE

(a) Find a quadratic equation of the form $y=c+bx+ax^2$ that goes through $(-2,20)$, $(1,5)$ and $(3,25)$ (7 marks)

(b) Using the Simpson's $\frac{1}{3}$ rule evaluate $I=\int_1^2 \frac{dx}{5+3x}$ with 8 subintervals. (7 marks)

(c) Using the data $\sin(0.1)=0.09983$ and $\sin(0.2)=0.19867$,

i) find an approximate value of $\sin(0.15)$

ii) find the relative error (6 marks)