



# TECHNICAL UNIVERSITY OF MOMBASA

---

FACULTY OF APPLIED & HEALTH SCIENCES

MATHEMATICS & PHYSICS DEPARTMENT

**UNIVERSITY EXAMINATION FOR:**

**BACHELOR OF TECHNOLOGY IN ENVIRONMENTAL PHYSICS &**

**RENEWABLE ENERGY**

**APS 4303: THERMAL PHYSICS II**

**END OF SEMESTER EXAMINATION**

**SERIES: MAY 2016**

**TIME: 2 HOURS**

**DATE: MAY 2016**

**Instructions to Candidates**

You should have the following for this examination

*-Answer Booklet, examination pass and student ID*

This paper consists of 4 questions.

**Do not write on the question paper. Answer question ONE (compulsory) and any other two questions.**

SECTION A (30POINTS)

QUESTION 1

(a) Explain the following terms

(i) Ensemble average

[3points]

(ii) Partition function

[3points]

(iii) Entropy

[3points]

(b) (i) Consider a one-particle system of two states, one of energy 0 and one of energy  $\varepsilon$ . The particles are in thermal equilibrium with a reservoir at temperature  $\tau$ . Compute the energy and heat capacity of the system as a function of then temperature  $\tau$ . [7points]

(ii) If we shift the zero energy and take the energies of the two states as  $-\frac{1}{2}\varepsilon$  and  $+\frac{1}{2}\varepsilon$ , compute the partition function and heat capacity of the system and find what the heat capacity looks like in conventional temperature system. [7points]

(c) Consider a model system with  $N_{\uparrow}$  spins up and  $N_{\downarrow}$  spins down. Let  $N = N_{\uparrow} + N_{\downarrow}$ ; the spin excess is  $2s = N_{\uparrow} - N_{\downarrow}$ . The entropy is given by (in Stirling's approximation)

$$\sigma(s) \approx -\left(\frac{1}{2}N + s\right) \log\left(\frac{1}{2} + \frac{s}{N}\right) - \left(\frac{1}{2}N - s\right) \log\left(\frac{1}{2} - \frac{s}{N}\right).$$

In a magnetic field B, what would be the free energy and what is the expression for the minimum energy? [7points]

## SECTION B

### QUESTION 2

(a) For a particle in a box the energy is given by

$$\varepsilon_n = \frac{\hbar^2}{2m} \left( \frac{\pi}{L} \right)^2 (n_x^2 + n_y^2 + n_z^2), \text{ where the letters have their usual meanings.}$$

(i) Give the expression for the partition function for this system. [4points]

(ii) What is the expression for the partition function if the spacing between adjacent energy is small in comparison with  $\tau$ . Use the formula,

$$\left( \int_0^\infty dn_x \exp(-\alpha^2 n_x^2) \right)^3 = \frac{\pi^{3/2}}{8\alpha^3} \quad [6points]$$

(b) The partition function of an ideal of N identical particles is given by

$$Z_N = \frac{1}{N!} (n_Q V)^N, \text{ where } n_Q = \left( \frac{M\tau}{2\pi\hbar^2} \right) \text{ and the letters have their usual}$$

meanings.

(i) Determine energy of the gas. [3points]

(ii) Determine the pressure of the gas. [3points]

liii) Determine the entropy of the system. [4points]

### QUESTION 3

(a) Give a brief description of the Debye model of heat capacity. [10points]

(b) In the Debye model of heat capacity the total energy is given by

$$U = \int_0^{n_D} dn \frac{\hbar\omega}{\exp(\hbar\omega/\tau) - 1}.$$

(i) Determine the total energy in the low temperature limit. [5points]

(ii) Determine the heat capacity  $C_V$  in the low temperature regime. [5points]

QUESTION 4

(a) Define the chemical potential of an ideal gas. [4points]

(b) The free energy of a monatomic gas is given by

$$F = -\tau \left[ \log Z_1 - \log N \right] \text{ where } Z_1 = n_Q V = \left( \frac{M\tau}{2\pi\hbar^2} \right)^{3/2} V$$

From this expression determine the chemical potential. [6points]

(c) The differential of entropy is given by  $d\sigma(U, V) = \left( \frac{\partial\sigma}{\partial U} \right)_V dU + \left( \frac{\partial\sigma}{\partial V} \right)_U dV$ .

If denote the independent values of  $dU$  by  $(\delta U)_n$  and  $dV$  by  $(\delta V)_n$  the entropy change will be zero.

(i) Determine the expression for the pressure in terms of  $\tau, \sigma, V$  with  $U$  kept constant. [6points]

(ii) From the expression for  $d\sigma$  obtain the thermodynamic identities. [4points]



