TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHELOR OF MATHEMATICS AND COMPUTER SCIENCE & BACHELOR OF SCIENCE IN STATISTICS AND COMPUTER SCIENCES

AMA 4209: **CALCULUS III**

END OF SEMESTER EXAMINATION

SERIES:APRIL2016

TIME:2HOURS

DATE: Pick DateMay2016

Instructions to Candidates
You should have the following for this examination
-*Answer Booklet, examination pass and student ID*
This paper consists of **FIVE** questions. Attempt question ONE (Compulsory) and any other TWO questions.
Do not write on the question paper.

QUESTION ONE (COMPULSORY 30MKS)

(a). Evaluate $\lim_{n\to\infty} \frac{3-e^n}{n^2}$ (3 marks)

(b). Determine whether the infinite series $\sum_{n=0}^{\infty} \frac{1}{n^2+3n+2}$ diverges or converges. If it converges, determine its sum. (5 marks)

(c).State any two conditions that a function f(x) must satisfy for it to be continuous at x = a (2marks)

(d). Compute $\int_0^1 \int_y^1 (3-x-y)\,dxdy$ (4 marks)

(e). Find the value(s) of c that satisfy the equation $f'(x) = \frac{f(b)-f(a)}{b-a}$ in the calculation of the mean value theorem for the function f(x) = $\sqrt{x-1}$ in the interval [1,3]. (5 marks) (f). State what is meant by the term centroid, hence determine the centroid of the triangular region R bounded by the x – axis, the y – axis and the line $y = 1 - \frac{1}{2}x$

(7 marks)

(g). Replace the polar equation $r^2 = 4r\cos\theta$ by equivalent Cartesian equations and name the graph. (4 marks)

QUESTION TWO (20MKS)

a) Find the volume of the solid formed when the plane figure bounded by $r = 2a\cos\theta$ and the vectors at $\theta = 0$ and $\theta = \frac{\pi}{2}$ rotates on the initial line. (5 marks)

b) A ball is dropped from a height of 6 meters and begins bouncing. The height of each bounce is three fourths the height of the previous bounce. Find the total vertical distance traveled by the ball up to rest. (5 marks)

c) Calculate $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at (1,2) for $f(x, y) = \sqrt{9 - x^2 - y^2}$ (4 marks)

d) Plot the graph of the polar equation $r = 1 + \cos\theta$, name the figure obtained hence find the area enclosed by the region obtained. (6marks)

QUESTION THREE (20MKS)

a).Use geometric series to express 0.103103103… as a ratio of two integers. (5 mark)

b).Determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{2n^3+100n^2+100}{\frac{1}{8}n^6-n+2}$ (5 marks)

c).Determine the first five terms of the Taylor's series generated by $f(x) = 1/x$ at x = 2

(10 marks)

QUESTION FOUR (20MKS)

a).State Rolle's theorem and justify it for the function $f(x) = x^3 - 4x$ on the interval [-2, 2]. (6 marks)

b).Given $z = \frac{2y}{y+cosx}$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ (6marks)

c).Evaluate $\lim_{x\to 1} \frac{x^3-1}{x-1}$. (3marks)

d).Determine whether the limits of f(x) exists if

$$f(x) = \begin{cases} 2x - x^3, & x < 1 \\ 2x^2 - 2, & x \geq 1 \end{cases}$$

(5marks)

QUESTION FIVE (20MKS)

a) Evaluate $\int_0^{\infty} \frac{dx}{x^2+4} dx$ (5 Marks)

b).Evaluate $\int_1^2 \int_y^{3y} (x+2y)\, dxdy$ (5 Marks)

c).Find the total derivative of z with respect to x if $z = f(x, y) = x^2 + 2xy + 4y^2$ where $y = e^{ax}$ (3 Marks)

d).Determine the radius of convergence of the power series $\sum_0^{\infty} n!\, x^n$ (7Marks).