FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BCE/BSEE/BSME

AMA4203/SMA 2272: STATISTICS

END OF SEMESTER EXAMINATION

SERIES: APRIL 2016

TIME: 2 HOURS

DATE: 13 May 2016

<u>**Instructions to Candidates**</u>
You should have the following for this examination
-*Answer Booklet, examination pass and student ID*
This paper consists of Choose No questions. Attempt Choose instruction.
Do not write on the question paper.

Question ONE (30 MarkS)

(a)   Define the following terms;
    (i)   Random experiment,   (1mark)
    (ii)   Random variable,   (1mark)
    (iii)   Sample space,   (1mark)
    (iv)   Conditional probability   (1mark)

(b)   A manufacturer of two types of food supplements wants to determine their efficacy in relieving the pain associated with arthritis. Efficacy data were collected from 100 patients as summarized in the following table;

		Supplement 1		
		High	Low	Total
Supplement2	High	70	9	79
	Low	16	5	21
	Total	86	14	100

Let A denote the event that supplement1 has high efficacy and B the event that supplement 2 has high efficacy. Determine the following:

    (i)   P(A)   (1mark)
    (ii)   P(B)   (1mark)
    (iii)   P(A\B)   (1mark)
    (iv)   P(B\A)   (1mark)

(v)   P(A∩B)   (1mark)

(vi)   Whether the events are independent   (1mark)

(c)   (i) Verify that the following function is a probability distribution function (2marks)

x	-1	0	1	2	3
p(x)	1/8	2/8	2/8	2/8	1/8

(ii)   Hence determine the probabilities;

$P(X \leq 3)$,   (2marks)

$P(-1 < X \leq 2)$ (2marks)

(d)   The number of telephone calls, X, received per minute at a switchboard has the

following probability distribution

x	10	11	12	13	14	15
p(x)	0.08	0.15	0.30	0.20	0.20	0.07

Determine;

(i)   The mean calls received per minute   (2marks)

(ii)   The variance of the number of calls received per minute (3marks)

(e)   Suppose the error in a controlled laboratory experiment is a random variable X
having the density function

$$f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2 \\ 0, & elsewhere \end{cases}$$

(i)   Verify that f(x) is a density function   (2 marks)

(ii)   Find E(X)   (3 marks)

(iii)   Find $E(X^2)$ (3 marks)

(iv)   Find var(X)   (1 mark)

Question TWO (20 MarkS)

Data indicating the number of candidates found cheating in last year's KCSE examinations for

a sample of 100 schools are summarized below.

Cheating candid	Numb. of schools
≥ 10 but < 20	16
≥ 20 but < 25	10
≥ 25 but < 30	20
≥ 30 but < 35	21
≥ 35 but < 40	14
≥ 40 but < 50	10
≥ 50 but < 70	4
Total	100

(a)   Draw a histogram depicting the above data.   (7 marks)

(b)   Estimate the mean and median of the data.   What do the data and your
         statistics indicate about the distribution of the number of cheating
         candidates?(6 marks)

(c)   Construct a 95% confidence interval for the mean number of candidates
        cheating in examinations in every school, stating any assumptions that you
        make.   (7 marks)

Question THREE (20 MarkS)

The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} kx^2(1-x)^2, & 0 \le x \le 1 \\ 0, & elsewhere \end{cases}$$

(a)   Find k and sketch the graph of f(x)   (7marks)

(b)   Find $E(X)$ and Var(X) and show that
$$P\left(X \le \frac{1}{3}\right) = \frac{17}{81}$$
                                                                                          (8marks)

(c)   A random sample of size 5 is taken from this distribution. Find, correct to
         4   decimal places, the probability that all 5 observations exceed 1/3. (3marks)

(d)   Find, correct to 4 decimal places, the variance of the mean of a random sample of
        size 5   (2 marks)

Question FOUR (20 MarkS)

A sample of 10 farmed tilapia fish was caught by a scientist whose interest was in the

the relationship between their length(x) and weight(y). The data are given in the table below.

Length (x)	387	366	329	293	273	268	294	198	185	169
Weight (y)	720	680	480	330	270	220	380	108	89	68

(a)   Plot the weights of the 10 tilapia fish (on the y or vertical axis) against
        the corresponding lengths (on the x or horizontal axis).   Does it appear
        appropriate to fit a straight line to these data?   (8 marks)

(b)   (i)   Calculate the least-squares estimates of the parameters β_0 and β_1 of the
                regression line $y = \beta_0 + \beta_1$

        (ii)   Comment on the appropriateness of the regression line estimated
                in part (i) as a model for the relationship between the weights and
                lengths of the particular variety of tilapia fish.   (8 marks)

(c)   Calculate and interpret the coefficient of determination.   (4 marks)

Question FIVE(20 Marks)

(a) The population of male students at TUM have height, H, distributed Normally with mean $160\,cm$ and standard deviation 4 cm, i.e. $N(160, 16)$. Find the proportion of the population whose heights are within one standard deviation of the mean. Find also the proportion of the population who are more than 168 cm tall.
                                                                              (5 marks)

(b) The University basket ball team (BBT) is restricted to persons who are more than 168 cm tall. and may be assumed to consist of a random sample of students satisfying this condition.
   Find;   (i)   the median height of members of the BBT,

            (ii)   the proportion of members of the BBT who are more than 170

                   cm tall.   (10 marks)

         (iii)   Assume that the mean and standard deviation of height among

                  members of the BBT are 169.5 cm and 1.352 cm respectively. Find

                  an approximate value for the probability that the mean height of
                  a random sample of 25 members of the BBT is more than 170 cm

                                                                              (5marks)