# FACULTY OF APPLIED AND HEALTH SCIENCES <br> DEPARTMENT OF MATHEMATICS \& PHYSICS <br> UNIVERSITY EXAMINATION FOR: <br> BSCE/BSEE/BSME 

AMA4203/SMA 2272: STATISTICS
END OF SEMESTER EXAMINATION
SERIES:APRIL2016
TIME:2HOURS
DATE:13May2016

## Instructions to Candidates

You should have the following for this examination
-Answer Booklet, examination pass and student ID
This paper consists of Choose No questions. AttemptChoose instruction.
Do not write on the question paper.
Question ONE (30 MarkS)
(a.) Define the following terms
i.) Equality of two sets A and B.
ii.) Complement of set A .
iii.) Union of set A and B.
(b.) A random variable X is said to be exponential distributed if it has a density function is in the form:

$$
f(x)=\left\{\begin{array}{l}
a e^{-a x}, \text { for } . . x \geq 0, a \geq 0 \\
0, . . \text { elsewhere }
\end{array}\right.
$$

Find
i.) an expression for $\mathrm{F}(\mathrm{x})$.
ii.) Sketch the pdf and PDF of X
iii.) Determine $\mathrm{P}(0<\mathrm{X} \leq 1)$ and $\mathrm{P}(\mathrm{X}>4)$.
(c.) Suppose that the number X of cracks per printed circuit board specimen used in the manufacture of a particular electro-mechanical device follows a Poisson probability distribution having an average of 2.5. Find
i.) the mean
ii.) the standard deviation
iii.) the probability that a randomly selected specimen has

- Exactly 5 cracks
- 2 or more cracks
(d.) Give a definition of the following terms
i.) Type I error
ii.) Type II error
iii.) Level of Significance
(e.) A random sample of 10 items is taken and is found to have a mean weight of 60 grams and a standard deviation of 12 grams. What is the mean weight of the population with a $95 \%$ confidence. ( 2 marks)
(f.) Give two properties of a good estimator.
(4 marks)


## Question TWO (20 MarkS)

a.) The pdf of $x$ is shown in Fig. 1.
i.) Determine the value of $\mathbf{A}$
ii.) Graph $F(x)$
iii.) Determine $P(x \geq 2 \mid x \geq 1)$
(2 marks)
(b.) The National Science Foundation in the USA reports that $70 \%$ of graduate students who earn PhD degrees in Engineering are foreign nationals. Consider the number Y of foreign students in a random sample of 25 Engineering students who earned their PhDs.
i.) Find $P(Y=10)$
ii.) Find $\mathrm{P}(\mathrm{Y} \leq 3)$
iii.) Find the mean $\mu$ and the standard deviation $\sigma$


## Fig 1.

## Question THREE (20 MarkS)

a.) It is known from previous data that the length of time in months between customers complaint about a certain product is a gamma distribution with $\alpha=2$ and $\beta=4$. Changes were made that involved tightening of quality control requirement. Following these changes, it took 20 months before the first complaint. Determine whether the quality control tightening was effective.
(b.) Based on extensive testing it is determined that the time Y in years before a major repair is required for a particular brand of refrigerator is characterised by the density function

$$
f(y)=\left\{\begin{array}{l}
\frac{1}{4} e^{-y / 4}, y \geq 0 \\
0 . . ., \text { elsewhere }
\end{array}\right.
$$

i.) The product is considered a bargain if it takes more than 6 years to require a major repair, determine whether this brand of refrigerator qualifies as a bargain.
ii.) Determine the probability that a major repair is required in the first year.

## Question FOUR(20 Marks)

Calculate,
(a.) Product moment coefficient of correlation.
(b.) Constants of the normal equations, $a_{0}$ and $a_{1}$, from the data below.
from the following data:

| $\mathbf{X}$ | 15 | 24 | 25 | 30 | 35 | 40 | 45 | 65 | 70 | 75 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Y | 60 | 45 | 50 | 35 | 42 | 46 | 28 | 20 | 22 | 15 |

## Question FIVE(20 Marks)

(a.) A Business System has two components A and B. The following events describe the states of the components:

A - First component is good; $\overline{\mathrm{A}}$ - First component is defective;
B - Second component is good; $\overline{\mathrm{B}}$ - second component is defective
Tests have shown that $\mathrm{P}(\mathrm{A})=0.8, \mathrm{P}(\mathrm{B} \mid \mathrm{A})=0.85, \mathrm{P}(\mathrm{B} \mid \overline{\mathrm{A}})=0.75$. Determine the following probabilities:-
i.) The second component is good.
ii.) At least one of the components is good.
iii.) The first component is good, given that the second is good.
iv.) The first component is good given that at most one component is good.
(b.) State whether the events represented by components A and B in (a.) are:
i.) Independent.
ii.) Mutually exclusive.
(c.) $A$ and $B$ are two events such that $P(A)=1 / 3, P(B)=2 / 9$ and $P(A \mid B)=1 / 2$.

Find $\mathrm{P}(\mathrm{B} \mid \overline{\mathrm{A}})$.
(d.) Determine the value of C which will make the function below be a probability distribution function for the random variable x :

$$
\begin{equation*}
f(x)=C\binom{2}{x}\binom{3}{3-x}, \text { for } . . x=0,1,2 \tag{2marks}
\end{equation*}
$$

