TECHNICAL UNIVERSITY OF MOMBASA

A Centre of Excellence

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATION FOR THE FIRST SEMESTER IN THE SECOND YEAR OF BACHELOR OF SCIENCE IN CIVIL AND MECHANICAL ENGINEERING

MAY 2016 SERIES EXAMINATION

UNIT CODE: SMA 2279

UNIT TITLE: LINEAR AND BOOLEAN ALGEBRA

TIME ALLOWED: 2HOURS

PAPER A

Instructions to Candidates:

You should have the following for this examination

- *Answer Booklet*
- *Scientific Calculator*

This paper consists of **FIVE** questions and **TWO** sections **A** and **B.**

Answer question **ONE** (**COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

This paper consists of **FOUR** printed pages.

SECTION A (COMPULSORY)

Question ONE (30 marks)

a. Prove that the diagonals of a rhombus are perpendicular. (5 marks)

b. Let P be the point $(2,1,1)$, Q be the point $(0,-2,3)$ and O be the origin. Find

i. The length of $\vec{PQ}$ (2 marks)

ii. $\underset{\sim}{a} \cdot \underset{\sim}{b}$ where $\underset{\sim}{a} = \vec{OP}$ and $\underset{\sim}{b} = \vec{OQ}$ (3 marks)

iii. The angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$ (3 marks)

c. Let $\underset{\sim}{a} = 2i + j - k$ and $\underset{\sim}{b} = 4i - 3j + 5k$. Find a unit vector $\hat{n}$ orthogonal to both $\underset{\sim}{a}$ and $\underset{\sim}{b}$. (5 marks)

d. Find the equation of the plane through $(-1, 2, 3)$ and perpendicular to the planes $2x - 3y + 4z = 1$ and $3x - 5y + 2z = 3$. (5 marks)

e. Use Cramer's rule to determine the solution to the system of equations:- (7 mark)

$$3x_1 - x_2 + 5x_3 = -2$$
$$-4x_1 + x_2 + 7x_3 = 10$$
$$2x_1 + 4x_2 - x_3 = 3$$

SECTION B

QUESTION TWO (20 MARKS)

(a) Use Gauss Seidel iterative technique to find the approximate solution correct to three decimal places:-

$$54x + y + z = 110$$
$$2x + 15y + 6z = 72$$
$$-x + 6y + 27z = 85$$

Take the initial approximate as $x_0 = y_0 = z_0 = 0$ for five iterates only. (9 marks)

(b) Find the solution of the following systems by Gauss-Jordan elimination method.(11 marks)

$$x - 3y + 4z = 12$$
$$2x - y - 2z = -1$$
$$5x - 2y - 3z = 3$$

QUESTION THREE (20 MARKS)

a. Find the inverse (A^{-1}) of the following matrix if it exists by row reduction method.

$$A = \begin{pmatrix} 2 & 1 & 1 \\ -5 & -3 & 0 \\ 1 & 1 & -1 \end{pmatrix}$$ (10 marks)

b. Reduce the following matrix into row echelon form: (5 marks)

$$A = \begin{pmatrix} 3 & 4 & -1 & 1 \\ 1 & -1 & 3 & 1 \\ 4 & -3 & 11 & 2 \end{pmatrix}$$

c. Compute AC and CA if possible given that

$$A = \begin{pmatrix} 1 & -3 & 0 & 4 \\ -2 & 5 & -8 & 9 \end{pmatrix}, \qquad C = \begin{pmatrix} 8 & 5 & 3 \\ -3 & 10 & 2 \\ 2 & 0 & -4 \\ -1 & -7 & 5 \end{pmatrix}$$ (5 marks)

QUESTION FOUR (20 MARKS)

(a) Given $A = \begin{pmatrix} 4 & 2 & 1 \\ -2 & -6 & 3 \\ -7 & 5 & 0 \end{pmatrix}$,

(i). Find the matrix of cofactors of A. (4 marks)
(ii). Find adjoint of matrix A (Adj A) (2 marks)
(iii). Calculate determinant of matrix A ($|A|$) (2 marks)
(iv).Using (ii) and (iii) above, Find A^{-1} (2 marks)

(b) Use Jacobi iterative technique to find the approximate solution correct to two decimal places:-

$$5x - y + z = 10$$
$$2x + 8y - z = 11$$
$$-x + y + 4z = 3$$

Take the initial approximate as $x_0 = y_0 = z_0 = 0$ for five iterates only. (10 marks)

QUESTION FIVE (20 MARKS)

a. Consider the matrix $A = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{pmatrix}$,

(i) Find the eigenvalues λ_i and eigenvectors V_j associated with the above matrix. (10 marks)

(ii) Show that the eigenvectors obtained in (i) above are linearly independent. (3 marks)

b. Complete the truth table of the following (7 mark)

p	q	⊔ p	⊔ q	p $\rightarrow$q	(p$\rightarrow$q) $\wedge$(q $\rightarrow$p)	(p$\leftrightarrow$ q)
T	T					
T	F					
F	T					
F	F					