



# TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

**UNIVERSITY EXAMINATION FOR:**

**BACHALOR OF MATHEMATICS AND COMPUTER SCIENCE**

**SMA4318: COMPLEX ANALYSIS 1**

**END OF SEMESTER EXAMINATION**

**SERIES: APRIL 2016**

**TIME: 2 HOURS**

**DATE: May 2016**

## Instructions to Candidates

You should have the following for this examination

-Answer Booklet, examination pass and student ID

This paper consists of **FIVE** questions. Attempt question ONE (Compulsory) and any other TWO questions.

**Do not write on the question paper.**

## Question ONE

(a) If  $z_1 = 3 - 4i$  and  $z_2 = 2 + 3i$  evaluate

(i)  $\overline{z_1 z_2}$  (4mrks)

(ii)  $\arg(z_2)$  (2mrks)

(b) Find the cuberoot of the complex number  $z = 4 + 4i$  (4mrks)

(c) Express the equation of the function  $f(z) = z^2$  in polar form (4mrks)

(d) Evaluate  $\lim_{z \rightarrow \infty} \frac{2z+1}{z+1}$  (4mrks)

(e) If  $f(z) = \frac{z^2-1}{2z+1}$ , Evaluate the derivative of  $f(z)$  (4mrks)

(f) Show that  $\sin(z) = \frac{e^{iz} - e^{-iz}}{2i}$  (4mrks)

- h) Determine the singular points of the function  $f(z) = \frac{2z+1}{(z^2-1)z}$  (4mrks)

### Question TWO

- (a) Find the Laurent series of the function  $f(z) = e^z$  (8mrks)
- (b) Find the residues of the function  $f(z) = \frac{z+1}{z^2(z^2+1)}$  in  $c$  where  $c: |z| \leq 1$  (7mrks)
- c) If  $z_0$  is any interior to the positive orientation of a simple close contour  $C$  and  $f(z)=1$  find  $\int_c \frac{f(z)}{z-z_0} dz$  (5mrks)

### Question THREE

- (a) Solve the equation  $e^z = -2$  (5mrks)
- (b) Prove that  $\ln z = \ln r + i\theta$  (5mrks)
- (c) Evaluate the differentiation of the function  $f(z) = (z^2 + 3z)(\cos z)^2$  (5mrks)
- (d) Evaluate  $\int_0^{\pi} e^{it} dt$  (5mrks)

### Question FOUR

- (a) Represent graphically the conformal mapping  $f(z) = Z^2$  for the line  $y=1$  (6 mrks)
- (b) Show that the function  $f(z) = \frac{z^2+4z}{z+4}$  is continuous at  $z=1$  (3mrks)
- (c) Show that the function  $f(z) = \bar{z}$  does not satisfies Cauchy riemann theorem (4mrks)
- (d) Show that the facton  $T(x,y) = e^{-y} \sin x$  is harmonic (4mrks)
- (e) Evaluate  $2^i$  (3mrks)

### Question FIVE

- (a) Use the Cauchy theorem to find  $\frac{dw}{dz}$  given that  $w = e^x (\cos y + i \sin y)$  (8mrks)
- (b) Find the harmonic conjugate of a harmonic function  $u(x,y) = y^3 - 3x^2 y$  hence find  $f(x,y)$  (8mrks)
- (c) If  $z = 2 + 4i$  find  $|\bar{z}|$  hence find the argument of  $z$  (2mrks)