TECHNICAL UNIVERSITY OF MOMBASA

A Centre of Excellence

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATION FOR THE FIRST SEMESTER IN THE SECOND YEAR OF BACHELOR OF SCIENCE IN CIVIL AND MECHANICAL ENGINEERING

MAY 2016 SERIES EXAMINATION

UNIT CODE: SMA 2279

UNIT TITLE: LINEAR AND BOOLEAN ALGEBRA

TIME ALLOWED: 2HOURS

Paper B

Instructions to Candidates:

You should have the following for this examination

- *Answer Booklet*
- *Scientific Calculator*

This paper consists of **FIVE** questions and **TWO** sections **A** and **B.**
Answer question **ONE** (**COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **FOUR** printed pages.

SECTION A (COMPULSORY)

Question ONE (30 marks)

a. Solve $\begin{aligned} 3x + y - z &= 3 \\ 2x - 8y + z &= -5 \\ x - 2y + 9z &= 8 \end{aligned}$ using Gauss-elimination method (7 marks)

b. Suppose that A is an invertible matrix and $|A| \neq 0$, prove that $\det(A^{-1}) = \dfrac{1}{\det(A)}$

(4 marks)

c. Find the vector equation of the line passing through the points $A(1,2,3)$ and $B(4,4,4)$ and find the coordinate of the point where this line meets the plane $z = 0$. (5 marks)

d. Find the equation of the plane passing through the point $(3,-1,7)$ and perpendicular to the vector $\hat{n} = (4,2,-5)$. (4 marks)

e. Reduce the following system of linear equations to row echelon form

$$\begin{aligned} x_1 - 2x_2 + 3x_3 &= -2 \\ -x_1 + x_2 - 2x_3 &= 3 \\ 2x_1 - x_2 + 3x_3 &= 1 \end{aligned}$$ (5 marks)

Hence solve the system.

f. Given that $B = \begin{pmatrix} 3 & -1 & 7 \\ 10 & 1 & -8 \\ -5 & 2 & 4 \end{pmatrix}$, $D = \begin{pmatrix} -1 & 4 & 9 \\ 6 & 2 & -1 \\ 7 & 4 & 7 \end{pmatrix}$, Compute BD and DB.

Is $BD = DB$? Give your comments. (5 marks)

SECTION B (Answer any TWO questions from this section)

Question TWO (20 marks)

a. Given $A = \begin{pmatrix} 2 & 1 & 2 \\ 3 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix}$,

(i). Find the matrix of cofactors of A. (4 marks)
(ii). Find Adj A (2 marks)
(iii) Calculate $|A|$ (2 marks)
(iv). Find A^{-1} (2 marks)

b. Use Cramer's rule to determine the solution to the system of equations:- (10 marks)

$$2x+3y-z=1$$
$$3x+5y+2z=8$$
$$x-2y-3z=-1$$

Question THREE (20 marks)

(a) Find the solution of the following systems by Gauss-Jordan elimination method.(10 marks)

$$x-3y+4z=0$$
$$2x-y-2z=5$$
$$5x-2y-3z=-8$$

(b) Use Jacobi iterative technique to find the approximate solution correct to two decimal places:-

$$5x-2y+3z=-1$$
$$-3x+9y+z=2$$
$$2x-y-7z=3$$

Take the initial approximate as $x_0=y_0=z_0=0$ for five iterates only. (10 marks)

Question FOUR (20 marks)

(a) Consider the matrix $A=\begin{pmatrix}-1 & -1 & 0\\ -4 & 3 & 0\\ 1 & 0 & 2\end{pmatrix}$,

(i) Find the eigenvalues λ_i and eigenvectors V_j associated with the above matrix.(10 marks)

(ii) Show that the eigenvectors obtained in (i) above are linearly independent.(4 marks)

(b) Evaluate the determinant (6 marks)

$$\begin{vmatrix} 1 & 6 & 0 & 1 \\ 2 & 11 & 0 & 13 \\ 4 & 5 & 7 & -2 \\ 0 & 1 & 0 & 1 \end{vmatrix}$$

Question FIVE (20 marks)

(a) Use Gauss Seidel iterative technique to find the approximate solution of the system of equations:-

$$\begin{aligned} 10x - 2y + z - w &= 3 \\ -2x + 10y - z + w &= 15 \\ -x - y + 10z - 2w &= 27 \\ -x - y - 2z + 10w &= -9 \end{aligned}$$

Take the initial approximate as $x_0 = y_0 = z_0 = w_0 = 0$ for five iterates only.(9 marks)

(b) Show that the LU decomposition method fails to solve the System of equation

$$A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 2 & 5 \\ 3 & 2 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$$

where the exact solution is $x_1 = 1, x_2 = 0, x_3 = -1$. (5 marks)

c. Complete the truth table of the following (6mark)

p	q	p → q	⊔ q	⊔ p	(q → p)	⊔ p → ⊔ q	⊔ q → ⊔ p
T	T						
T	F						
F	T						
F	F						