TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF ENGINEERING & TECHNOLOGY

MECHANICAL ENGINEERING

UNIVERSITY EXAMINATION FOR:

BACHELOR OF SCIENCE

EMG 2414: Numerical Methods for Engineers

END OF SEMESTER EXAMINATION

SERIES:APRIL 2016

TIME: 2 HOURS

DATE: 2016

Instructions to Candidates

You should have the following for this examination
-Answer Booklet, examination pass and student ID
This paper consists of Choose No questions. AttemptChoose instruction.
Do not write on the question paper.

Question ONE

(a) Using Cramer's Rule solve the linear system of equations

$$2x + y = 7$$
$$3x - 4y = 5$$

(5 Marks)

(b) Solve the following system of equations using Gauss elimination method

$$2x_2 + x_3 = -8$$
$$x_1 - 2x_2 - x_3 = 0$$
$$-x_1 + x_2 + 2x_3 = 3$$

(5 Marks)

(c) Let $A = \begin{bmatrix} 7 & 10 \\ 1 & -2 \end{bmatrix}$, find the Eigen values and the corresponding Eigen vectors of A

(5 Marks)

(d) Find $f'(3)$ using the Newton's backward difference formula

x	1	1.5	2	2.5	3
$f(x)$	-1.5	-2.875	-3.5	-2.625	0.5

(5 marks)

(e) Obtain a divided difference table for the following data

x	-1	0	2	3
$f(x)$	-8	3	1	12

(5 marks)

(f) Approximate the definite integral

$\int_0^1 \sqrt{1+x^2}\,dx$ Using the trapezoidal rule correct to 4 decimal places using $\Delta x = 0.2$

(5 marks)

Question TWO

(a) Consider $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

Find i) A^2,

ii) Eigen values and corresponding Eigen vectors of A^2, . **(7 Marks)**

(b) Use Euler's method to numerically integrate $\frac{dy}{dx} = -2x^3 + 12x^2 - 20x + 8.5$ from $x = 0$ to $x = 1$ with a step size of 0.5 the initial condition at $x = 0$, is $y = 1$ **(7 Marks)**

(c) Using forward difference formula, estimate $f'(x)$ of $f(x) = -0.1x^4 - 0.15x^3 - 0.5x^2 - 0.25x + 1.2$ at $x = 0.5$ using a step size of $h = 0.5$

(6 Marks)

Question THREE

(a) If $y = x^3 - x^2 + x - 1$, calculate the values of y for $x = 0,1,2,3,4,5$ and form the Backward difference table. (7 Marks)

(b) The table below shows data for $f(x) = 0.2 + 25x^2 - 200x^2 + 675x^3 - 900x^4 + 400x^5$ with unequally spaced values.

x	$f(x)$
0	0.2
0.12	1.309729
0.22	1.305241
0.4	2.456
0.54	3.507297
0.7	2.363
0.8	0.232

Using the trapezoidal rule for unequally spaced values, find $\int_0^{0.8} f(x)dx$ **(7 marks)**

(c) Using Gaussian Elimination, Solve the system of linear equations

$$8x_1 + 5x_2 + 11x_3 = 30$$
$$-x_1 - 4x_2 + 2x_3 = 3$$
$$2x_1 - x_2 + 5x_3 = 12$$

(6 marks)

Question FOUR

(a) Show that the second divided difference of $f(x) = \frac{1}{x}$, Using the points (a,b,c) is $\frac{1}{abc}$

(7 marks)

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(b) (i) State Lagrange's formula of interpolation using unequal intervals.

(2 marks)

(ii) Using Lagrange's interpolation formula, find the value of y corresponding to x=3 from the table below. **(5 marks)**

x	0	1	4
y	1	3	105

(c) Compute $f'(2.0)$ using backward difference table from the following tabular data.

x	1.4	1.6	1.8	2.0
$f(x)$	4.0552	4.9530	6.0496	7.3981

(6 marks)

Question FIVE

(a) (i) State the formula of trapezoidal rule **(2 marks)**

(ii) A curve passes through $\{(0,1),(0.25,0.9412),(0.5,0.8),(0.75,0.64),(1,0.5)\}$

Find $\int_0^1 f(x)dx$ by trapezoidal rule. **(6 marks)**

(iii) How can the accuracy of the trapezoidal rule be increased? **(2 marks)**

(b) Find an approximate value of $\log_e 5$ by approximating $\int_0^5 \frac{1}{4x+5}dx$ using Simpson's $\frac{1}{3}$ rule of integration using $n = 10$ equal sub- intervals. **(10 marks)**