TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BTIT

SMA 2230: PROBABILITY & STATISTICS II

END OF SEMESTER EXAMINATION

SERIES:APRIL2016

TIME:2HOURS

DATE:20May2016

Instructions to Candidates

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

This paper consists of Choose No questions. AttemptChoose instruction.

Do not write on the question paper.

Question ONE (30 MarkS)

(a) The random variable Y follows a binomial distribution with probability mass function given by

$$f(y) = \binom{n}{y} p^y (1-p)^{n-y}, \qquad y = 0,1 \dots, n; 0 < p < 1$$

(i) Write down the mean and variance of Y. (2 marks)

(ii) An intelligence test consists of 48 multiple-choice questions. For each question, four possible answers are presented but only one is correct. If a student answers all the questions independently by random guesswork, what will be the distribution of the number of questions he gets right? (2 marks)

(b) Apple juice is dispensed by a machine into cartons. The nominal volume of apple juice in a carton is 1 litre (1000 ml). The actual volumes of juice put into the cartons can be regarded as being independently normally distributed, with mean set at 1010 ml and standard deviation 8 ml.

Find the proportion, in a long run of production, of cartons containing less than the nominal volume (4 marks)

(c) State the Normal approximation to the binomial distribution, indicating the conditions under which it is valid. In 50 firings, a surface-to-air missile (SAM) is successful in hitting its target in 30 cases. Obtain an approximate 95% confidence interval for the probability, p say, that a given missile hits its target. (8 marks)

(d) Let X have a binomial pmf

$$f(x)=\begin{cases}\frac{3!}{x!(3-x)!}\left(\frac{2}{3}\right)^{x}\left(\frac{1}{3}\right)^{3-x}, & x=0,1,2,3\\ 0, & elsewhere\end{cases}$$

Find the distribution of $Y=X^2$ (5 marks)

(e) The continuous random variable T has probability density function (pdf) f(t) given by

$$f(t)=\lambda e^{-\lambda t}, \quad t>0;\ \lambda>0.$$

(i) Show that the cumulative distribution function is given by $F(t)=1-e^{-\lambda t}, \quad t>0;\ \lambda>0$. (2 marks)

(ii) Deduce that $P(a<T\le b)=e^{-\lambda a}-e^{-\lambda b}, \quad 0<a<b.$ (2 marks)

(f) The random variable X has probability density function f(x) given by

$$f(x)=\frac{k}{x^{k+1}}, \quad x\ge 1,\ k>0.$$

Find the median and the lower and upper quartiles of X and deduce the semi-interquartile range of X. (5 marks)

Question TWO (20 MarkS)

(a) The population of male students at TUM have height, H, distributed Normally with mean 160 cm and standard deviation 4 cm, i.e. N(160, 16). Find the proportion of the population whose heights are within one standard deviation of the mean. Find also the proportion of the population who are more than 168 cm tall. (5 marks)

(b) The University basket ball team (BBT) is restricted to persons who are more than 168 cm tall.

and may be assumed to consist of a random sample of students satisfying this condition.

Find:

(i) the median height of members of the BBT, (5 marks)

(ii) the proportion of members of the BBT who are more than 170 cm tall. (5marks)

(iii) Assume that the mean and standard deviation of height among members of the BBT are 169.5 cm and 1.352 cm respectively. Find an approximate value for the probability that the mean height of a random sample of 25 members of the BBT is more than 170 cm (5mark)

Question THREE (20 MarkS)

Fatal accidents occur at random at a known 'black spot', following a Poisson process with mean 4 per year.

(a) Draw a diagram of the probability mass function of X, the actual annual number of fatal accidents, and calculate the probabilities of the following events.

(i) In a given year there is at most one fatal accident.

(ii) In a given 6-month period there are no fatal accidents.

(iii) In a given 18-month period there are no fatal accidents. (10 marks)

(b) The random variable X follows the exponential distribution with rate parameter λ, so that the probability density function (pdf) of X is given by

$$f(x) = \lambda e^{-\lambda x}, \quad x > 0, \ \lambda > 0.$$

Show that the moment generating function of X, $M_X(t)$ say, is given by

$$M_X(t) = \left(1 - \frac{t}{\lambda}\right)^{-1}, \ t < \lambda,$$

and hence show that the mean and variance of X are given by $\frac{1}{\lambda}$ and $\frac{1}{\lambda^2}$ respectively (10 marks)

Question FOUR(20 Marks)

The random variable X has probability density function f(x) given by

$$f(x) = kx^2(1-x), \quad 0 \le x \le 1.$$

(a) Show that k = 12. (2 marks)
(b) Show that the mode of X is at x = 2/ 3 and draw a graph of f(x). (5 marks)
(c) Find the mean and variance of X. (5 marks)
(d) Find the cumulative distribution function of X and obtain the probability that X lies within one standard deviation of its mean (8 marks)

Question FIVE(20 Marks)

(a) The random variable X follows the Poisson distribution with probability mass function

$$f(x) = e^{-\lambda}\frac{\lambda^x}{x!}, \quad x = 0, 1, 2, \dots .$$

(i) Find the moment generating function of X. (2 marks)
(ii) Hence or otherwise show that the mean and variance of X are both equal to λ (5 marks)
(iii) State the Poisson approximation to the binomial distribution, indicating the circumstances in which it is appropriate (2mark)

(b) A civil servant calculates weekly social security payments for unemployed adults. These payments vary according to claimants' circumstances, and errors may occur. Over a long period of time, the probability of a wrong calculation has been found to be 0.0075. Find to 4 decimal places the exact probability that a sample of 200 contains

(i) 1 wrong calculation, (2 marks)
(ii) (ii) 4 wrong calculations. (2 marks)

(c) Repeat part (ii) using the Poisson approximation to the binomial distribution. In each case find to two significant figures the percentage error in the Poisson calculation. Comment briefly on your results. (7 marks)