



A Centre of Excellence

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

MAY 2016 SERIES EXAMINATION

**UNIT CODE: AMA 4438 UNIT TITLE: APPLICATIONS OF
FLUID MECHANICS**

SPECIAL/SUPPLEMENTARY EXAMINATION

TIME ALLOWED: 2HOURS

INSTRUCTION TO CANDIDATES:

You should have the following for this examination

- Mathematical tables
- Scientific Calculator

This paper consists of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

QUESTION ONE (30 MARKS)

- a. (i) Distinguish between an ordinary point and a singular point of a differential equation (2 marks)
- (ii) Determine the singular points of the equation
 $(x^2 - 9) \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + (x + 3)y = 0$ (4 marks)
- b. Define pinch effect in relation to plasma. Hence show that
 $p - p_0 = \frac{I\mu}{\pi a^4} (a^2 - R^2)$ Where p is the pressure outside the plasma $R \leq a$ is the radius (6 mks)
- c. Find the Laplace transform of $7e^{2t} + 9e^{-2t} + 5\cos t + 7t^3 + 5\sin 3t + 2$ (5 marks)
- d. Determine the equation of the path of the river flowing that satisfies the equation
 $xy \frac{dy}{dx} = x^2 - 1$ and when it passes through a point (1, 2) on the Cartesian plane. (5 marks)
- e. State the Hamilton's variation principle for a steady state flow of fluid and define all the symbols you have used. (4 marks)
- f. State four ways in which water obtains its ions (4 marks)

QUESTION TWO (20 MARKS)

- a. Determine a solution $u(x,y)$ of the Laplace equation $\frac{d^2y}{dx^2} + \frac{d^2y}{dy^2} = 0$ subject to the b.c
 $U = 0$ when $x = 0$ $u = 0$ when $x = \pi$
 $U \rightarrow 0$ when $y \rightarrow 0$ $u = 3$ when $y = 0$ (6 marks)
- b. State Luke's Variation principle for a constant equilibrium depth and non constant equilibrium depth for the given Ω . outlining the equation of conservation of mass for irrotational flow, the dynamic free surface condition and the condition of zero flow through the bed. (4 marks)
- c. Define the following terms according to the knowledge of finite elements
- The element domain (2 marks)
 - The space of shape function (2 marks)
 - A nodal basis (6 marks)

QUESTION THREE (20 MARKS)

- a. The efficiency η of a fan depends on the density ρ and the dynamic viscosity μ of the fluid, the angular velocity ω , diameter D of the rotor and the discharge Q . express η in terms of dimensionless parameters (5 marks)
- b. Briefly discuss the following aspects ground water flow
- i. Ground water (3 marks)
 - ii. Properties of porous media (3 marks)
 - iii. Infiltration (3 marks)
- c. Locate and classify the singular points of the differential equation
 $(x^2 - 8x) \frac{dy}{dx} + (x + 2) \frac{dy}{dx} + y = 0$ (3 marks)
- d. Solve the differential equation below by Laplace transform
 $\frac{d^2y}{dx^2} - 3 \frac{dy}{dx} = 9$ Given that when $x=0$ $y=0$ and $\frac{dy}{dx} = 0$ (3 marks)

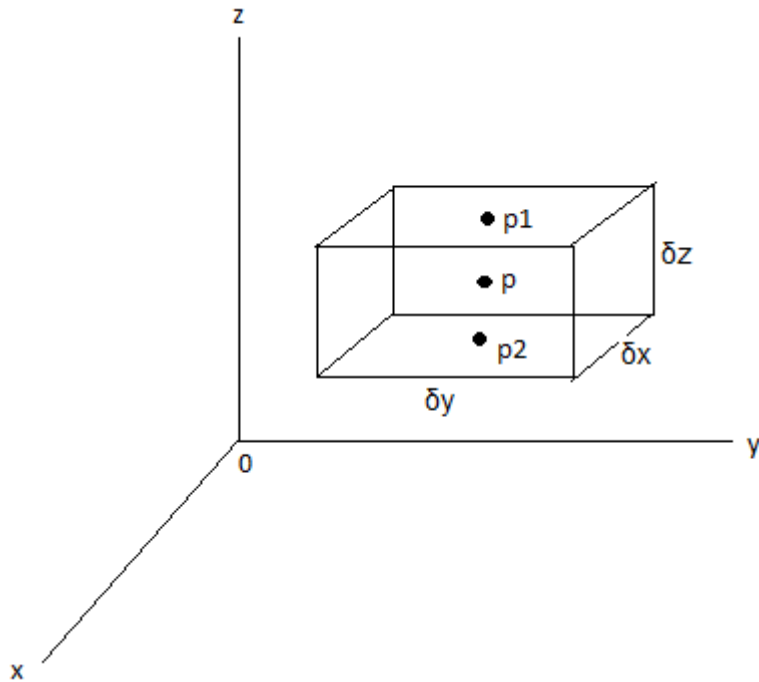
QUESTION FOUR (20 MARKS)

- a. State two factors that affects viscosity (2 marks)
- b. A plate 0.05mm distant from a fixed plate moves at 1.2m/s and requires a force of 2.2N/m² to maintain this speed. Find the viscosity of the fluid between the plate (4 marks)
- c. From the Navier Stokes Vector Equation
$$\rho \frac{dy}{dx} = F - \nabla p + \frac{1}{3} \rho v \nabla \nabla \cdot v + \rho v \nabla^2 v$$

Where ρ = fluid density
 V = fluid velocity
 F = Body force per unit volume
 P = Fluid pressure
 v = Kinematic coefficient of viscosity
 ρv = Coefficient of viscosity
Derive the hydrodynamic equation of motion of a conducting fluid. (7 marks)
- d. Discuss the following
- i. Properties of porous media (4 marks)
 - ii. Properties of water (3 marks)

QUESTION FIVE (20 MARKS)

- a. Consider the motion of a small rectangular parallel piped of viscous fluid, its centre being $p(x, y, z)$ and its edges of length $\delta x, \delta y, \delta z$ parallel to fixed Cartesian axes.



Taking the mass of the fluid element to be constant and the element to move along the fluid. Show that there is a relationship between the Cartesian components

(6 marks)

- b. Show that for a conducting fluid at rest , the charge decays very rapidly (6 marks)
- c. A coil spring lies along the helix. $r = (\cos 4t)i + (\sin 4t)j + tk, 0 \leq t \leq 2\pi$. The spring's density is a constant $\delta=1$. Find spring's mass and spring's moment of inertia and radius of gyration about the z axis (3 marks)
- d. State and prove the Alfvén's theorem for a perfect conducting fluid (5 marks)

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