TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

DIPLOMA IN MECHANICAL, ELECTRICAL, BUILDING AND CIVIL

ENGINEERING YEAR I SEMESTER II

AMA 2151: ENGINEERING MATHEMATICS II

END OF SEMESTER EXAMINATION

SERIES: APRIL 2016

TIME: 2HOURS

DATE: Pick DateMay2016

Instructions to Candidates
You should have the following for this examination
-Answer Booklet, examination pass and student IDMathematical table, calculator, no mobile phone
This paper consists of **FIVE** questions. Attempt question ONE (Compulsory) and any other TWO questions.
Do not write on the question paper.

Question ONE

(a)   Differentiate from first principles, the functions

        (i) $y = \tan x$ (5 marks)

        (ii) $y = 1/x$ (4 marks)

  (b)   Obtain the following integrals

        (i) $\int \frac{2x+3}{\sqrt{(2x^2+6x-1)}}dx$ (4 marks)

        (ii) $\int \tan\theta\, d\theta$ (3 marks)

(c)   The distance x metres moved by a body in t-seconds is given by
$$x = 3t^3 - 1\tfrac{1}{2}t^2 + 2t + 5$$

Determine:
(i)   Its velocity after 4 minutes   (2marks)
(ii)   The value of t when the body comes to rest   (3 marks)
(iii)   The acceleration after 2 seconds.   (2 marks)

(d)   Determine where the function
$$h(t) = \frac{4t + 10}{t^2 - 2t - 15} \text{ is not continuous.}$$ (3 marks)

(e)   Determine all the numbers C which satisfy the conclusions of the mean value theorem for the function.
$$f(x) = x^3 + 2x^2 - x \text{ on } (-2,2)$$ (4 marks)

Q.2   (a)   Differentiate the following functions
(i) $y = x^{\frac{1}{2}} Sin3x$ (3 marks)

(iii) $y = \dfrac{x2}{\sqrt{x+1}}$ (5 marks)

(b)   Show that if $y = \log_e x; \dfrac{dy}{dx} = \dfrac{1}{x}$ (3marks)

(c)   A piece of wire 4.0m long is cut into two parts, one of which is bent into a square and the other bent into a circle.
Find the radius of the circle if the sum of their areas is minimum.   (9 marks)

Q.3   (a)   Determine the turning points for the function and distinguish between them
$$y = 2x^3 + 7x^2 + 4x - 3$$ (8 marks)

(b)   An inverted right circular cone of vertical angle 120° is collecting water from a tap at steady rate of $18\pi cm^3$/minute.

(i)   Determine the depth of water after 12 minutes.   (3 marks)
(ii)   The rate of increase of the depth after the 12 minutes.   (4 marks)

(c)   A rectangular sheet of metal which measures 24cm by 16cm has squares removed from each of the four corners so that an open box may be formed. Find the maximum volume of the box.   (5 marks)

Q.4   (a)   Obtain (i) $\int \dfrac{5x^3 - 3x^2 + 41x - 64}{(x^2 + 6)(x - 1)^2}dx$ (10 marks)

(ii) $\int x^2 \bullet Log_e\, xdx\, dx$ (4 marks)

(b)   Determine the area enclosed between
$y = 4 - x^2$ and $y = x^2 - 2x$ (6 marks)

Q.5   (a)   Sketch the curves for
(i) $y = Sin\, h\, x$ (2 marks)
(ii) $y = Cosh\, x$ (2 marks)

(b)   Differentiate
$y = \tan hx$ (3 marks)

(c)   Show that
$y = Cosh\, xCoshy - SinhxSinhy = Cosh(x - y)$ (6 marks)

(d)   Solve for x in the equation
$3Cosh\, 2x + 5Coshx = 22$ (7 marks)

(a)   Differentiate from first principles, the functions
(i) $y = \tan x$ (5 marks)
(ii) $y = \dfrac{1}{x}$ (4 marks)

(b)   Obtain the following integrals

(iv) $\int \dfrac{2x + 3}{\sqrt{(2x^2 + 6x - 1)}}dx$ (4 marks)

(v) $\int \tan \theta\, d\theta$ (3 marks)

(c)   The distance x metres moved by a body in t-seconds is given by
$x = 3t^3 - 1\frac{1}{2}t^2 + 2t + 5$

Determine:

(iv)   Its velocity after 4 minutes   (2marks)

(v)   The value of t when the body comes to rest   (3 marks)

(vi)   The acceleration after 2 seconds.   (2 marks)

(d)   Determine where the function

$$h(t) = \frac{4t + 10}{t^2 - 2t - 15}$$ is not continuous.   (3 marks)

(e)   Determine all the numbers C which satisfy the conclusions of the mean value theorem for the function.

$$f(x) = x^3 + 2x^2 - x \text{ on } (-2,2)$$ (4 marks)

(a)   Differentiate from first principles, the functions

(i) $y = \tan x$ (5 marks)

(ii) $y = \frac{1}{x}$ (4 marks)

(b)   Obtain the following integrals

(vi) $\int \frac{2x + 3}{\sqrt{(2x^2 + 6x - 1)}} dx$ (4 marks)

(vii) $\int \tan \theta \, d\theta$ (3 marks)

(c)   The distance x metres moved by a body in t-seconds is given by

$$x = 3t^3 - 1\frac{1}{2}t^2 + 2t + 5$$

Determine:

(vii)   Its velocity after 4 minutes   (2marks)

(viii)   The value of t when the body comes to rest   (3 marks)

(ix)   The acceleration after 2 seconds.   (2 marks)

(d)   Determine where the function

$$h(t) = \frac{4t + 10}{t^2 - 2t - 15}$$ is not continuous.   (3 marks)

(e)   Determine all the numbers C which satisfy the conclusions of the mean value theorem for the function.

$$f(x) = x^3 + 2x^2 - x \text{ on } (-2,2)$$ (4 marks)

Q.2   (a)   Differentiate the following functions

(i) $y = x^{\frac{1}{2}} Sin3x$ (3 marks)

(viii) $y = \dfrac{x2}{\sqrt{x+1}}$ (5 marks)

(b)   Show that if $y = \log_e x;\ \dfrac{dy}{dx} = \dfrac{1}{x}$ (3marks)

(c)   A piece of wire 4.0m long is cut into two parts, one of which is bent into a square and the other bent into a circle.
Find the radius of the circle if the sum of their areas is minimum.   (9 marks)

Q.3   (a)   Determine the turning points for the function and distinguish between them
$$y = 2x^3 + 7x^2 + 4x - 3$$ (8 marks)

(b)   An inverted right circular cone of vertical angle 120° is collecting water from a tap at steady rate of $18\pi cm^3$/minute.

(iii)   Determine the depth of water after 12 minutes.   (3 marks)
(iv)   The rate of increase of the depth after the 12 minutes.   (4 marks)

(c)   A rectangular sheet of metal which measures 24cm by 16cm has squares removed from each of the four corners so that an open box may be formed. Find the maximum volume of the box.   (5 marks)

Q.4   (a)   Obtain (i) $\displaystyle\int \dfrac{5x^3 - 3x^2 + 41x - 64}{(x^2 + 6)(x - 1)^2}dx$ (10 marks)

(ii) $\displaystyle\int x^2 \bullet Log_e xdx\, dx$ (4 marks)

(b)   Determine the area enclosed between
$$y = 4 - x^2 \quad \text{and} \quad y = x^2 - 2x$$ (6 marks)

Q.5   (a)   Sketch the curves for
(i) $y = Sin\,h\,x$ (2 marks)
(ii) $y = Cosh\,x$ (2 marks)

(b)   Differentiate
$$y = \tan hx$$ (3 marks)

(c)   Show that

$$y = Cosh\,xCoshy - SinhxSinhy = Cosh(x-y)$$ (6 marks)

(d)   Solve for x in the equation
$$3Cosh\,2x + 5Coshx = 22$$ (7 marks)

(a)   Differentiate the following functions

   (i) $y = x^{1/2} Sin3x$ (3 marks)

   (ix) $y = \dfrac{x2}{\sqrt{x+1}}$ (5 marks)

(b)   Show that if $y = \log_e x;\ \dfrac{dy}{dx} = \dfrac{1}{x}$ (3marks)

(c)   A piece of wire 4.0m long is cut into two parts, one of which is bent into a square
        and the other bent into a circle.
        Find the radius of the circle if the sum of their areas is minimum. (9 marks)

Question THREE

(a)   Determine the turning points for the function and distinguish between them
$$y = 2x^3 + 7x^2 + 4x - 3$$ (8 marks)

(b)   An inverted right circular cone of vertical angle 120° is collecting water from a
        tap at steady rate of $18\pi cm^3$/minute.

   (v)   Determine the depth of water after 12 minutes. (3 marks)
   (vi)   The rate of increase of the depth after the 12 minutes. (4 marks)

(c)   A rectangular sheet of metal which measures 24cm by 16cm has squares removed
        from each of the four corners so that an open box may be formed. Find the maximum volume of
        the box. (5 marks)

Question FOUR

(a)   Obtain (i) $\displaystyle\int \dfrac{5x^3 - 3x^2 + 41x - 64}{(x^2 + 6)(x-1)^2}dx$ (10 marks)

          (ii) $\displaystyle\int x^2 \bullet Log_e xdx\,dx$ (4 marks)

(b)   Determine the area enclosed between
$$y = 4 - x^2 \quad \text{and} \quad y = x^2 - 2x \qquad \text{(6 marks)}$$

Question FIVE

(a)   Sketch the curves for
   (i) $y = \operatorname{Sin} h \, x$ (2 marks)
   (ii) $y = \operatorname{Cosh} x$ (2 marks)

(b)   Differentiate
$$y = \tan hx \qquad \text{(3 marks)}$$

(c)   Show that
$$y = \operatorname{Cosh} x \operatorname{Cosh} y - \operatorname{Sinh} x \operatorname{Sinh} y = \operatorname{Cosh}(x - y) \qquad \text{(6 marks)}$$

(d)   Solve for x in the equation
$$3\operatorname{Cosh} 2x + 5\operatorname{Cosh} x = 22 \qquad \text{(7 marks)}$$